Abstract—Unbalance is a damaging and costly phenomenon for both network operators and end users. Network operators are increasingly looking at ways in which they can intelligently monitor their exposure to unbalance and its related problems. This paper presents initial results obtained with a new methodology for estimation of the voltage unbalance factor at any busbar in a distribution network. The proposed methodology utilizes distribution system state estimation to estimate the statistical properties of voltage unbalance factor. Both real and pseudo measurements are used to estimate the voltage unbalance factor and the corresponding error in the estimate. The methodology is validated, at this stage, on a simple generic unbalanced power system network using real data from UK distribution networks. It is shown to yield promising results across a series of unbalance simulations.

Index Terms—Voltage unbalance, distribution system state estimation, state estimation, voltage unbalance factor

I. INTRODUCTION

Unbalance is an important aspect of the wider topic of power quality. Unbalance can damage assets in a power network, increase losses and significantly affect the operation of sensitive customer equipment.

Unbalance typically originates from either asymmetries in network topology or unbalanced loads. Network asymmetries originate from dissimilar inter-phase coupling of impedances, asymmetrical transformer windings and open wye or open delta transformer banks [1, 2]. Loading asymmetries are caused when the three phase distribution circuit is unevenly loaded. This may originate from a variety of sources including heavily loaded single phase loads, two phase loads, single phase traction loads [3] and single phase rural distribution networks[2].

Single phase loads are not affected by unbalance unless the unbalance causes over or under voltages which exceed statutory limits. The most sensitive loads to unbalance are three phase motors [4]. Under unbalanced conditions, three phase motors will experience increased heating, noise, vibration and a loss of efficiency. Induction motors and adjustable speed drives (ASDs) are also particularly vulnerable to unbalance [2]. Unbalanced conditions also contribute to increased power system losses, and increased heating of utility assets, which are both problems for distribution network operators (DNOs).

Balancing loads is challenging due to continual hourly fluctuations in the power demand from loads connected to distribution circuits. For example, the power demand from large industrial loads such as arc furnaces and adjustable speed drives varies continuously. Increasing numbers of single phase switched mode power supplies also causes unbalanced conditions to be created within the network.

Finally, ever increasing connections of distributed generators to the network, many of which are single phase and operate either in stochastic or intermittent mode, further aggravates the voltage balance in the network.

The current lack of monitoring in modern distribution networks keeps the unbalance problem relatively hidden from DNOs. Unbalance issues are usually tackled on a case by case basis. Indeed, the large numbers of single phase residential loads in typical distribution networks means that unbalance can exist predominantly undetected. However, the steady increase of distributed generators which are sensitive to unbalanced power system conditions is likely to project unbalance, and other power quality issues, to the forefront of a DNOs network reinforcement strategy.

The drive towards making power grids more flexible, observable and controllable, i.e., towards smart grids, is likely to result in a significant increase in the number of monitors distributed throughout the network. It is becoming increasingly important for DNOs to consider how they will intelligently process and interpret large amounts of information gathered from meters within distribution networks.

Distribution system state estimation (DSSE) [5-8] is one emerging technique which is capable of processing information from a partially monitored distribution network. DSSE uses both real time and also pseudo measurements to estimate the state of the system and has been performed on both three phase[5] and single phase[8] models of power system networks. DSSE typically uses the weighted least squares (WLS) statistical technique in a similar manner to transmission system state estimation.

This paper presents a new methodology using DSSE to determine the voltage unbalance factor (VUF)[9]. In an advancement of existing research, the asymptotic covariance matrix from DSSE is used to calculate the statistical error of the VUF estimates. The technique is shown to be a reliable
and viable of estimating the amount of unbalance in a distribution network.

II. STATE ESTIMATION

State estimation algorithms were traditionally designed to work with transmission networks, where there is a high redundancy of telemetered measurements available. In distribution networks, there is often little or no redundancy of measurements.

The state estimation problem is defined mathematically by

\[ e = z - H(x) \]  \hspace{1cm} (1)

Where \( z \) is a vector of measurements, \( H(x) \) is a non-linear set of equations that describes the true state of the power system with state variables \( x \) and \( e \) is a vector of the errors between the observed measurements and true state of the system. \( e \sim N(0, \Sigma) \), where \( \Sigma \) is the covariance matrix of the errors in the measurement variables \( e \).

In this research, the weighted least squares (WLS) state estimation technique was used. The objective of the WLS estimation problem is shown in (2).

\[ \min_{x} [z - H(x)]^T \Sigma^{-1} [z - H(x)] \] \hspace{1cm} (2)

The state estimation equations can be solved iteratively, using the Newton-Raphson method. The update equations are defined in equation (3).

\[ x_{k+1} = x_k + (H_x \Sigma^{-1} H_x)^{-1} H_x \Sigma^{-1} [z - H(x_k)] \] \hspace{1cm} (3)

\[ H_x \frac{\partial H(x)}{\partial x} \] \hspace{1cm} (4)

Where \( x_{k+1} \) is the estimate for the state variables at the \((k+1)\)th iteration. \( H_x \) is the Jacobian matrix.

A. Three Phase State Estimation

The three phase WLS state estimator is formulated to solve for the state vector \( x \). \( x \) can be defined as a set of three phase voltage magnitudes and phase angles at each bus in the network[10]. The measurement vector \( z \) can be similarly defined as a vector of three phase active and reactive power demands.

In this research, there are 5 different types of measurements that could be taken at buses \( i \) and \( j \) throughout the network: real \((P_i^p)\) and reactive power \((Q_i^p)\), voltage \((V_i^p)\), real power flow \((P_{ij}^p)\) and reactive power flow \((Q_{ij}^p)\) in a line. Each of these measurements can be formulated in terms of the complex voltages at a bus in rectangular coordinates. The state estimation equations are solved in rectangular coordinates so that the distribution of the voltage unbalance factors can be found using a simple linear transformation (shown in equation (16) later).

The equations for the real \((P_i^p)\) and imaginary \((Q_i^p)\) components of the voltages at each bus \( i \) for the \( p \)th phase are shown below.

\[ P_i^p = \sum_{k=1}^{N} \sum_{m=\{a,b,c\}} G_{ik}^{pm} (P_{i,k}^m - y_{ik} P_{k}^m) + B_{ik}^{pm} (V_{i,k}^m - y_{ik} V_{k}^m) \] \hspace{1cm} (5)

\[ Q_i^p = \sum_{k=1}^{N} \sum_{m=\{a,b,c\}} G_{ik}^{pm} (Q_{i,k}^m + y_{ik} P_{k}^m) - B_{ik}^{pm} (V_{i,k}^m - y_{ik} V_{k}^m) \] \hspace{1cm} (6)

\[ P_{ij}^p = -\sum_{m=\{a,b,c\}} c_{ij}^{pm} (P_{i}^p + y_{ij} P_{j}^p) + G_{ij}^{pm} (P_{i}^m + y_{ij} P_{j}^m) - B_{ij}^{pm} (V_{i,k}^m - y_{ij} V_{j}^m) \] \hspace{1cm} (7)

\[ Q_{ij}^p = \sum_{m=\{a,b,c\}} B_{ij}^{pm} (P_{i}^m + y_{ij} P_{j}^m) + G_{ij}^{pm} (P_{i}^m + y_{ij} P_{j}^m) + B_{ij}^{pm} (V_{i,k}^m - y_{ij} V_{j}^m) \] \hspace{1cm} (8)

Where the \( p \)th phase is either, \( a, b \) or \( c, G_{ij}^{pm} \) is real \( i, j \)th element of the three phase admittance matrix, and \( B_{ij}^{pm} \) is the imaginary \( i, j \)th element of the three phase admittance matrix.

If there are no voltage measurements available at any buses other than the slack bus, the Jacobian matrix in rectangular coordinates has constant curvature (i.e. the Hessian \( \frac{\partial^2 H(x)}{\partial x^2} \) is made up entirely of constants).

B. Errors in State Estimation

The measurement vector is assumed to be distributed normally with variances given in the covariance matrix \( \Sigma \). An estimate for the state variable covariance matrix[11] is shown in (12).

\[ E[x - \hat{x}] = 0 \] \hspace{1cm} (10)

\[ \text{cov}(x - \hat{x}) = G^{-1} \] \hspace{1cm} (11)

\[ G^{-1} = (H_x \Sigma^{-1} H_x)^{-1} \] \hspace{1cm} (12)

\[ (x - \hat{x}) \sim N(0, G^{-1}) \] \hspace{1cm} (13)

Where \( H_x \) is the Jacobian found from the last iteration of the Newton-Raphson method.

Equations (10) to (13) describe the distribution of the errors in the state variables found through state estimation. The equations can therefore be used to predict the expected variation in the estimates found through DSSE.

III. VOLTAGE UNBALANCE

The voltage unbalance factor (VUF) is defined as the ratio of the negative sequence voltage over the positive sequence voltage[9] as shown by (14).

\[ \text{VUF} = 100 \times \frac{\text{negative sequence voltage}}{\text{positive sequence voltage}} \] \hspace{1cm} (14)

DSSE can be used to calculate the VUF by first calculating voltage magnitudes and angles throughout the network and then calculating the negative and positive sequence voltages. However, by considering the state estimation problem in rectangular (rather than the polar) coordinates, an expression for the distribution of the VUF can be exactly obtained.
A. State Estimation of Voltage Unbalance

The state variables found from the solution to the state estimation equations can be straightforwardly transformed into the positive and negative sequence voltages using a linear transformation. A matrix \( Z \) is defined to convert the three phase voltages at a bus into positive and negative sequence voltages[12]. The VUF at each of the buses in the network can then be found. Equation (16) shows this transformation.

\[
x_i^1 = \begin{bmatrix} y_{ia}^b & y_{ia}^c & y_{ic}^a & y_{ic}^b & y_{ic}^c \end{bmatrix}^T \quad (15)
\]

\[
x_i^{0.1^2} = Zx_i \quad (16)
\]

Where \( x_i \) is an estimated set of complex voltages (in rectangular coordinates) at the \( i \)th bus and \( x_i^{0.1^2} \) is a set of positive, negative and zero sequence voltages (in rectangular coordinates) at the \( i \)th bus.

The asymptotic covariance matrix for the state estimates can be transformed linearly by \( Z \) as shown in (17).

\[
G^{-0.1^2} = ZG^{-1}Z^T \quad (17)
\]

The operation in (17) is linear, and therefore the errors in the variables \( x_i^{0.1^2} \) are distributed as \( N(0, G^{-0.1^2}) \).

An estimate for the distribution of the VUF at the \( i \)th bus can be obtained as shown in (18).

\[
VUF = 100 \times \sqrt{\frac{(y_{ic}^{(2)})^2 + (y_{ic}^{(2)})^2}{(y_{ic}^{(1)})^2 + (y_{ic}^{(2)})^2}} \quad (18)
\]

Where \( y_{ic}^{(1)} \) is an estimate (obtained from state estimation and (16)) for the real component of the positive sequence voltage at the \( i \)th bus, \( y_{ic}^{(2)} \) is the complex component of the positive sequence voltage at the \( i \)th bus, and the superscript (2) indicates negative sequence.

Each of the variables in (18) has a multi-variate normal distribution which can be estimated from (17). (18) can therefore be used to predict the expected variation in \( \bar{VUF} \).

It is clear from (18), that \( \bar{VUF} \) will not be normally distributed. However, in the simulations presented in Section VI it is shown that the distribution of \( \bar{VUF} \) can be approximated to a normal distribution and that this distribution can be used to obtain the standard deviation and intervals for the \( \bar{VUF} \) estimates.

IV. MODELING LOAD PROFILES OF DISTRIBUTION NETWORKS

The input to a DSSE model is a set of power and voltage measurements. In traditional state estimation, the errors in the measurements (\( e \) in equation (1)) are assumed to be distributed normally.

A typical pseudo measurement is formulated from an estimate for the real or reactive power demand at a bus in a network. The real power demand at a bus follows a profile similar to the one shown in Fig. 1 below.

Fig. 1. The estimated apparent power and true power at a bus as estimated by a linear regression model of a domestic unrestricted customer. The dashed lines represent boundaries taken to be \( \pm 3 \) standard deviations (3SD)

In order to obtain statistical estimates from the state estimation equations the load profile does not need to follow any form of standard distribution. However, to obtain statistical insight into the results of state estimation, it is useful if the expected error in pseudo measurements is approximately normally distributed.

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An estimate for the distribution of the VUF at the \( i \)th bus can be obtained as shown in (18).

\[
VUF = 100 \times \sqrt{\frac{(y_{ic}^{(2)})^2 + (y_{ic}^{(2)})^2}{(y_{ic}^{(1)})^2 + (y_{ic}^{(2)})^2}} \quad (18)
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Where \( y_{ic}^{(1)} \) is an estimate (obtained from state estimation and (16)) for the real component of the positive sequence voltage at the \( i \)th bus, \( y_{ic}^{(2)} \) is the complex component of the positive sequence voltage at the \( i \)th bus, and the superscript (2) indicates negative sequence.

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Fig. 1 shows that the load profile does not follow any typical statistical distribution function. It has been suggested that the profile follows an approximate log normal or beta distribution function[7, 14]. It has also been suggested (in [8]) that the load profiles can be split into a sum of normal distributions by utilizing a Gaussian mixture model.

In order to obtain statistical estimates from the state estimation equations the load profile does not need to follow any form of standard distribution. However, to obtain statistical insight into the results of state estimation, it is useful if the expected error in pseudo measurements is approximately normally distributed.

The error in a pseudo measurement will depend on many external factors including time of day, temperature, type of load and weather patterns. Load profile estimates can be obtained by using a number of methods (such as those discussed in [18]). A simple method of load profile estimation is to create a linear regression model that aims to predict the load consumed based on a series of regressor variables.

Predicting the load for each of the 4 types of customers can be modeled as a function of the following 4 variables: \( x_1 \) - Time in \( \frac{1}{2} \) hourly intervals; \( x_2 \) - Sunset; \( x_3 \) - Noon effective temperature and \( x_4 \) - Day of the week.

To simplify the regression, the analysis can be conducted considering each of the days in the variable \( x_4 \) as a constant. The dependent variable in the analysis is the single phase
equivalent of the total real power demand for a given consumer type $j$.

The regression model can be described using the following equation:

$$P_{10}^j = X\hat{\beta} + r_i \quad \text{(19)}$$

$$\hat{P}_{10}^j = x\hat{\beta} \quad \text{(20)}$$

Where $\hat{\beta}$ is a set of coefficient estimates that minimize the sum of squared residuals $r_i$. $X$ is an $n \times 3$ matrix of independent variables for each of the $n$ observations. $P_{10}^j$ is a set of $n$ observed real power demands for the $j$th consumer type; D/U, D/E, I or C. $\hat{P}_{10}^j$ is the least squares estimate for the $j$th consumer type based on a single set of specific independent variables, $x$.

If the residuals ($r_i$) are normally distributed, then the constructed model ($x\hat{\beta}$) is an estimate for the power at a bus ($\hat{P}_{10}^j$) which has the desired characteristics for input into a distribution state estimation model. In other words, the measurement variables have errors which are normally distributed.

Fig. 2 shows the distribution of the residuals produced when building a 10 fold cross validation[15] linear regression model for a domestic unrestricted set of customers. The right hand plot confirms that the residuals are indeed normally distributed. Fig. 1 shows an estimated reconstructed profile during a 24 hour period with the regression model shown above.

The Anderson-Darling statistic[16, 17] can also be used to determine if the hypothesis that the samples were drawn from a normal distribution can be rejected. At the 0.1% significance level, the Anderson-Darling statistic cannot reject the hypothesis that the distributions are normally distributed.

![Figure 2](attachment:figure2.png)

**Figure 2.** The distribution of the residuals found when estimating real power demand for a domestic unrestricted customer across 2000 separate hours. Data obtained from an (undisclosed) UK distribution network record and recorded during 2008 and 2009.

It should be noted that much more sophisticated load profile models have been developed which utilize neural networks, fuzzy logic, support vector machines and other artificial intelligence techniques[18]. However, regardless of the modeling technique used, the output requirements are the same. If the model can estimate load whilst producing a set of normally distributed residuals, then the normality assumption for the errors in the measurement variables in the state estimation equations holds.

B. Modeling the Power Demand in 3 Phase Loads

For each consumer type, the single phase equivalent power consumed (or generated) at a bus is a summation of the power in each of the three phases. Without prior knowledge of unbalance, it is reasonable to assume that the best estimate for the powers consumed within each of the three phases would be a third of the power consumed in total. If the power in each of the three phases is represented as a $3 \times 1$ vector of three random variables ($P_{30}^j$) then the expected value of this vector is described by equation Error! Reference source not found.: 

$$E(P_{30}^j) = \begin{bmatrix} \bar{P}_a^j \\ \bar{P}_b^j \\ \bar{P}_c^j \end{bmatrix} = \begin{bmatrix} \bar{P}_{10}^j/3 \\ \bar{P}_{10}^j/3 \\ \bar{P}_{10}^j/3 \end{bmatrix} = \bar{P}_{30}^j \quad \text{(21)}$$

C. Modeling the Expected Demand at a Generic Bus

The customers connected to a busbar are often a mixture of industrial, commercial and residential customers. When more than one customer is connected to a single busbar, an estimate for the load can be obtained by combining the estimated loads of each of the customer classes attached to a busbar[8].

The method outlined in [8] creates a load profile estimate in 3 stages. Firstly, four scaling parameters are computed which scale each of the four generic load profiles by the maximum demand for each customer at the busbar. This is shown in equation as $a_i^j$ in equation (22).

$$a_i^j = \frac{p_{i,10,max}}{\max_{\tau}(P_{i,10,\tau}(\tau))} \quad \text{(22)}$$

Where $p_{i,10,max}$ is the maximum power for the $j$th customer class (D/U, D/E, I or C) at the $i$th bus, and $\max_{\tau}(P_{i,10,\tau}(\tau))$ is the maximum value of the $j$th generic load profile. Both maximums are taken over a 1 year period.

The four scaling parameters are then used to linearly combine the modeled variables to generate an estimate for the total power consumed at a bus. This is shown in equation (23).

$$P_{10}^j(t) = \sum_{i=1}^{N_j} a_i^j \bar{P}_{10}^j(t) \quad \text{(23)}$$

Where $N_j$ is the number of profile classes (4 in this study). $\bar{P}_{10}^j(t)$ is an estimate for the power of the $j$th consumer class obtained from the linear regression model. It can be modeled as a function of time, since the regressor variable $x_i$ represents time in half hourly intervals. $P_{10}^j(t)$ is an estimate for the power consumed at the $i$th bus.

The reactive power can be derived from the active power estimate by assuming a typical power factor for each of the consumer classes[8].

$$Q_{10}^j(t) = \sum_{j=1}^{N_j} a_j^j \bar{P}_{10}^j(t) \tan (\Theta_j) \quad \text{(24)}$$
The power factors used in this study were the same as those proposed in [8]; 0.95, 0.99, 0.98 and 0.90 lagging for D/U, D/E, I and C respectively. Three phase estimates for the consumed power can be computed by using equation Error! Reference source not found..

V. SIMULATION METHOD

The DSSE state estimation method for voltage unbalance factor was tested on a simple generic power system network. The network used was a modified version of the 4 bus network used in [12]. The generators in the network were modified slightly and a generator was only connected to bus 1. The impedances in the network were left unaltered. The network used was a modified version of the 4 bus network used in [12]. The generators in the network were modified slightly and a generator was only connected to bus 1. The network diagram is shown in Fig. 3.

![Diagram of the 4 bus network modified from [12]](Image)

Fig. 3. The 4 bus network (modified from [12]) used in this study.

The VUF was estimated at each of the buses in the network across three separate loading scenarios. In all the situations, the slack bus is assumed to provide the network with a balanced set of voltage phasors precisely 120° out of phase with one another. There is a monitor placed at bus 1, which measures real and reactive power consumed, along with the real and reactive power transmitted to both bus 2 and bus 3. All the measurements taken at bus 1 are for all three phases.

A. State Variables & Measurements

The aim of the state estimator is to estimate the variables \( \gamma_p^p \) and \( \gamma_q^p \) at buses 2, 3 and 4 for all phases. The state variable vector (\( \mathbf{x} \)) therefore has 18 elements.

The measurements at bus 1 are telemetered measurements with a standard deviation of 1% of the true value of the power[8]. Measurements at bus 2, 3 and 4 are all pseudo measurements. These were assumed to have a standard deviation equal to 7% of the true value of the power. This value was derived from empirical evidence from tests (shown in Fig. 2) where the accuracy of the linear regression model is found to have a standard deviation of approximately 7%. The figure also corresponds closely with the value used in [8] for pseudo measurements.

The type of customers attached to each of the busbars was varied throughout the simulations.

B. Case studies

Three case studies were conducted at this stage of research. In each case, load profiles for the various customer types were obtained from the UK GDS [13] dataset over a typical 24 hour period. In all cases, each bus was normalized using (22) to ensure it was loaded with the same maximum power consumption.

The case studies conducted were as follows:

1) CS1 - Residential customers were connected to busbars 2, 3 and 4. The load on phase A at each bus was persistently simulated to be 20% higher than a third of the average single phase equivalent power. The load on phases B and C were simulated to be 90% of the average single phase equivalent power.

2) CS2 - Residential customers were connected to busbars 2 and 3. An industrial customer was connected to bus 4. The load on phase A at buses 2 and 3 was persistently simulated to be 20% higher than a third of the average single phase equivalent power, in the same manner as CS1. The load at bus 4 was assumed to be balanced.

3) CS3 - The same customer configuration was used as in CS2. However, an extra meter was connected to monitor the real and reactive power demand at bus 4.

For each of the case studies, the variation in voltage unbalance factor was plotted over a typical 24 hour period. At each ½ hourly time step, 10,000 samples of 4 multi-variate normally distributed random variables were generated (as shown in equation (17)). The 4 sets of 10,000 samples were then fed into equation (18) to generate the estimated distribution of the VUF. The distribution of the VUF was shown to be approximately normal. The standard deviation and mean of the VUF distribution were then used to show the expected variation of the estimated VUF to within ±3 standard deviations (3SD).

VI. RESULTS

Prior to testing the performance of the DSSE on each of the CSs, an initial test was conducted to test the assumption that the estimated distribution of the VUF was normally distributed. This test was carried out with the network loaded as in CS1.

The true distribution of the VUF was determined by conducting a Monte Carlo simulation. A power flow was initially run on the network with the load set to the maximum experienced during the typical 24 hour period. 1000 input measurements were generated for each of the real and pseudo measurements with a mean equal to the true value (found from the power flow) and variance of either 1% or 7% for real and pseudo measurements respectively.

![Distribution of VUF](Image)

Fig. 4. Top plot: the true distribution of the VUF found when the network was loaded as in simulation 1, with real and pseudo measurement errors of 1% and 7%.
and 7% respectively. Bottom plot: the estimated distribution of the VUF at bus 4 found by utilizing the asymptotic covariance matrix of equation (17).

The true distribution of the VUF found from the Monte Carlo simulation is shown in Fig. 4. This is very similar to the estimated distribution of the VUF found from the asymptotic covariance matrix of equation (17) (bottom plot of Fig. 4).

Fig. 5. The normal probability plot of the true VUF at bus 4. The histogram of the distribution is shown in the upper plot of Fig. 4.

The assumption that the VUF is approximately distributed as a normal distribution can be shown with a normal probability plot. The normal probability plot of the true VUF (upper plot of Fig. 4) is shown in Fig. 5. The normal probability plot shows a strong correlation indicating that the approximation that the distribution is normally distributed is valid.

A. Case study 1

Fig. 6 shows the results from CS1. It shows that the unbalance state estimation technique tracks the true voltage unbalance factor closely when the load is persistently increased on phase A at buses 2, 3 and 4. It also shows that the distribution of the VUF at bus 4 is wider than at either bus 2 or bus 3. This is because the error in the estimate at bus 4 is generated largely from the error in the pseudo estimate for the load at 4, whereas at buses 2 and 3 the power line measurements increase the level of accuracy of the resulting VUF estimate.

B. Case study 2

Fig. 7 shows the results of changing the load type at bus 4. The load consumption at bus 4 is now entirely balanced. However, the state estimator apportions the load evenly onto each of the 3 buses. This causes the dashed band of the VUF to be on average higher than the true value of the VUF. However, when the industrial load is high, and the residential load is light (during the night), the true VUF is more centrally aligned to the estimated region.

Fig. 7. The true VUF (green solid line) and the estimated VUF +3 standard deviations (blue dashed line). The results are shown over a typical 24 hour period for bus 2 (top plot), bus 3 (middle plot) and bus 4 (bottom plot).

C. Case study 3

In CS3 extra monitor was placed at bus 4 to monitor the real and reactive power demand of the system. Fig. 8 shows that the accuracy of the state estimator is much improved when an extra monitor is added to network at bus 4.

Fig. 8. The true VUF (green solid line) and the estimated VUF +3 standard deviations (blue dashed line). The results are shown over a typical 24 hour period for bus 2 (top plot), bus 3 (middle plot) and bus 4 (bottom plot) with an extra monitor placed at bus 4.

VII. CONCLUSION

The paper presented a new methodology and initial results to estimate voltage unbalance factor using distribution system state estimation. The methodology used a modified version of three phase state estimation to successfully estimate the statistical properties of the voltage unbalance factor.

The methodology was tested, at this stage, on a simple generic power system network and shown to yield promising results. It was able to correctly identify the level of unbalance
throughout the network and obtain statistical distributions for the estimates.

Based on these initial results it is anticipated that the methodology could be used to estimate the level of voltage unbalance within large, realistic size, distribution networks, and even lead to identification of the sources of unbalance within the network.

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