Decaying DC Offset Removal Operator Using Mathematical Morphology For Phasor Measurement

J. Buse, D.Y. Shi, T.Y. Ji, and Q.H. Wu, Senior Member, IEEE

Abstract—Phasor measurement is an essential component of protection relays and phasor measurement units. Phasor measurement as performed by the Fourier Transform is adversely affected by the decaying DC offset which occurs following a fault. Hence an efficient mathematical pre-processor is proposed to remove this offset in a half cycle plus an additional sample. Following this it is possible to use a new phasor measurement technique which can estimate the phasor using a quarter cycle window. The results show that the mathematical preprocessor can virtually remove the DC offset, and that the new phasor measurement technique out performs the Fourier Transform due to its significantly shorter delay.

Index Terms—Mathematical morphology, DC offset, DFT, protective relays, phasor measurement.

I. INTRODUCTION

Phasor measurement is the measurement of the magnitude and phase angle of a sinusoidal component, in particular the fundamental component. It is used in protection relays and phasor measurement units. Following a fault, a decaying DC offset is often produced in the current waveform due to the inductive nature of power lines preventing an instantaneous change in current. This decaying DC offset covers the whole spectrum of frequencies, hence it “contaminates” the phasor measurement. Phasor measurement is typically performed using the Fourier Transform and the decaying DC offset results in a series of decaying oscillations. This is of particular importance for protection relays as it limits the accuracy and response time of the relay. Furthermore, if the DC offset is removed, it paves the way for using shorter-windowed phasor measurement techniques.

A number of techniques have been presented in literature to reduce this effect. The work can be broadly group into two categories—the methods including a model of the DC offset in the phasor measurement, and the methods which calculate and remove the DC offset prior to the phasor measurement. Originally in the days of electromechanical relays, mimic circuits were developed to mimic or model the DC offset and hence cancel it. Later digital mimic filters were developed for microprocessor-based relays, based on the analogue mimic circuits [1]. Least square fitting filters have also been applied to model the DC offset by a Taylor series expansion [2]. The mimic filters have an inherent dependence on the DC offset time constant, being designed for a time constant in the middle of the expected range, and the least square fitting techniques have limited time constant representation, dependent on the Taylor series expansion applied. Furthermore, they both tend to amplify the high-frequency components, causing a reduction in noise immunity.

In the second category there are a number of different methods to estimate the DC offset. The work done by Sidhu and Balamourougan uses a filter set at a frequency way above the cut of frequency, from which to estimate the parameters of the DC offset [3]. They expanded the method, for application as a half cycle phasor estimator, and to improve the real time performance in [4] and [5], respectively. In [6] a method which uses up to three successive DFTs to calculate the parameters of the DC offset was proposed. The method can be based on the HCDFT instead of the FCDFT depending on the nature of the signal, in particular if there are no even harmonics. In [7] and [8] the fact that the sum of the signal over a period should be zero, but not the DC offset is used to estimate the parameters of the DC offset. In [9] the even and odd parts of the DFT are used to estimate the parameters of the DC offset. These methods can perform well though the computation can be high in estimating the parameters, as such the method [5] uses a lookup table to simplify the calculations.

This paper proposes a mathematical morphological pre-processor to remove the DC offset, following which a new phasor measurement technique involving the embedding of the signal in the phase space is used in comparison with the Fourier Transform. The use of mathematical morphology creates a simple and efficient operator that is ideal for hardware implementation. Mathematical morphology is based on simple operators such as addition and subtraction which can be implemented in parallel in hardware. The new phasor measurement technique also allows the calculation of the phasor with a delay as short as a quarter cycle.

II. PRINCIPLES FOR DC OFFSET REMOVAL

A. Mathematical Morphology

Mathematical Morphology (MM) is a relatively new technique to process images and signals based on the shape information. It originated from the work done by Serra and Matheron in 1964 and has been applied to a wide range of applications, including noise removal and object extraction. It has been used in medical applications such as ECG feature extraction, machine vision applications and power system applications including protective relaying. MM was originally applied to binary images, but has been extended to greyscale images and signals. The principle is that there are two sets—a larger set the image or signal which is denoted by S and a smaller set the structuring element (SE) which is denoted by B. The structuring element is then moved through the image/signal pixel by pixel performing the required operation.
The basic operators are erosion and dilation, the greyscale version of which are defined as:

\[
(S \ominus B)(x) = \min_{(x+v) \in S, v \in B} \{S(x+v) - B(v)\} \\
(S \oplus B)(x) = \max_{(x-v) \in S, v \in B} \{S(x-v) + B(v)\}
\]

where \(x\) corresponds to a pixel in the image/signal and \(v\) a pixel in the structuring element. Based on these two operators, the opening and closing are defined as:

\[
S \circ B = (S \ominus B) \oplus B \\
S \bullet B = (S \oplus B) \ominus B.
\]

B. MM DC Offset Removal Operator

A MM opening defined by (3) removes any peaks with a duration less than the SE length. Hence, if the SE length is equal to a half cycle, the peak half of the signal will be removed. Similarly a mathematical closing defined by (4) removes any troughs whose duration is less than the SE length. Therefore, if a mathematical opening followed by a closing is performed, the peaks and troughs will be removed leaving the DC offset. The effect of an opening, closing, and an opening followed by a closing is illustrated in Fig. 1. The delays between the different operators have not been shown, so as to clearly display the result of each operator. The delay for the opening followed by a closing is three quarters of a cycle, while the delay for just an opening or closing is half a cycle.

To reduce the window length, only the opening is performed during the peak half cycle, and only the closing is performed during the trough half cycle. The resulting operator is given as follows

\[
D = \begin{cases} 
S \circ B & \text{if peak half cycle} \\
S \bullet B & \text{if trough half cycle}.
\end{cases}
\]

Furthermore, if the symmetry of the signal is considered at the peak or trough point, the reflection of the signal can be performed. Hence, the half cycle window of data needed for the operator can be produced with a quarter cycle of data [10]. This is illustrated in Fig. 2. To improve accuracy it was decided to sacrifice the reduced delay and to average the results from the two quarter cycles of data, which is also inline with the maximum delay caused by the handling of the phase shift. Thus, this method has a constant delay of a half cycle. In the following discussion, the peak or trough point is referred to as the peak.

Impulse noise, as caused by ADC misread, interference, etc, can cause the peak detection to mislocate the peak. To solve this problem, the input signal is firstly processed by a morphological filter to remove this noise. The filter used is the morphological DC offset removal operator, giving a total delay of a half cycle and a sample.

C. Handling the Phase Shift at Fault Onset

When a fault occurs, there is a change of impedance, which will introduce a phase shift. The phase shift will change the width of the period during the fault onset. The effects of the phase shift can be divided into three categories as illustrated in Fig. 3.

1) The fault instant is following the peak in the current half cycle.
2) The fault instant is proceeding the peak in the current half cycle.
3) The fault instant is at the same instant as the peak in the current half cycle.

In case 1 the DC offset value should be held, for the duration of the half cycle. For case 2 the DC offset value should be held until the fault time, and after the fault time the value from the second quarter of the half period should be used. In case 3 since the value of the fault instant is the same as the peak instant, the remaining part of the half cycle must be less than or equal to a quarter cycle. Hence the structuring
If $\delta = T/4$, then the $x$ and $y$ coordinates are given by:

$$x = A_1 \cos(\omega t + \varphi)$$

$$y = A_1 \sin(\omega t + \varphi).$$

These are orthogonal components representing the real and imaginary components of the signal respectively. Therefore, the magnitude is simply given by:

$$A_1 = \sqrt{x^2 + y^2}$$

and the angle is given by:

$$\theta = \arctan \left( \frac{y}{x} \right).$$

The phase can also be given relative to an arbitrary sine wave:

$$\phi = \arctan \left( \frac{y}{x} \right) - \text{mod}(\omega t_n, 2\pi).$$

where mod(.) denotes the modulo operation. This method allows the phasor to be calculated with a very short delay of one quarter cycle, as opposed to a full cycle delay for the full cycle discrete Fourier Transform (FCDFT), or a half cycle delay for the half cycle discrete Fourier Transform (HCDFT). Generally a post-processor is required, to reduce the influences of noise and other disturbances. A median filter was chosen, due to its good edge preservation characteristics, allowing the output to change rapidly following a fault. The window length used is a half cycle. To allow the phasor measurement to respond even faster to a sudden change in magnitude following a fault, the filter window is cleared when a fault occurs. This is given by the following equation:

$$A(x) = \text{median}_{(x+v) \in S,v \in B} \{S(x+v)\}$$

where

$$B = \{b_1, b_2, \ldots, b_{l-1}, b_l \}$$

if $f_t \leq x - l$ or $f_t \geq x$

$$b_f, b_{f+1}, \ldots, b_{l-1}, b_l \}$$

if $x - l \leq f_t \leq x$.

where $l = T/2$ and $f_p = x - f_t$. $f_t$ is the fault time, and $f_p$ is the position of the fault in the window.

### E. Fault Onset Detection

To handle the phase shift, the fault start point needs to be found; furthermore, the fault start point is needed to reset the medium filter. The fault start point is found by considering the change in phase difference between the signal and a nominal 50Hz signal. This can be derived from (15) as follows:

$$\Delta \phi(x) = \phi(x) - \phi(x - 1).$$

If the change in phase difference is greater than a threshold, the fault start is detected. To prevent the fault start being detected multiple times, the change in phase difference has to settle below a lower threshold before it can be re-detected.
F. Frequency Compensation

When the frequency changes, the length of a period becomes longer or shorter, and therefore, the length of the structuring element is not quite the right size. If the frequency increases, the length of the period becomes shorter; hence, the structuring element is longer than half a cycle. This means the DC offset value is lower in the peak half cycle and higher in the trough half cycle, causing the magnitude of the signal to be increased. If the frequency is decreased the converse is true.

To compensate for the change in frequency, firstly the frequency deviation is measured. The frequency deviation is the average rate of change of phase difference between the signal and a nominal 50Hz signal derived from (15) as follows. The rate of change of phase difference is given by:

$$\Delta \phi_2(x) = \phi(x) - \phi(x - T)$$

and the average rate of change of phase difference is given by:

$$\overline{\Delta \phi_2(x)} = \frac{1}{T} \sum_{i=x-T}^{x} \Delta \phi_2(i).$$

Secondly the magnitude calculated by the phasor measurement is multiplied by twice the proportion change in frequency. This is due to the increase in magnitude in the peak half cycle added to the increase in magnitude in the trough half cycle.

$$A_c = A_1 \times \frac{1}{1 + (\Delta \phi_2(x)/300) \times 2}$$

Thirdly the phasor measurement is “tuned” to 50Hz, which causes oscillations in the output. To prevent this the phasor measurement can be “retuned” by altering the $\theta$ in (10). When using the assumption $\delta = T/4$ to calculate the phase, the value of $\delta$ can be updated accordingly assuming a significantly high sampling frequency is used.

III. Performance Evaluation

The evaluation is performed in Matlab using data generated from PSCAD and data generated from the equations. The simulated PSCAD data is generated using the model shown in Fig. 4. The fault is located equidistant between the two buses. The transmission line is also modelled, by the lumped parameter model shown in Fig. 5, which leads to the following simple equations for current before and after the fault respectively:

$$I_a = A_a \sin(\omega t + \varphi)$$

$$I_b = A_b \sin(\omega t + \varphi + \beta) + B e^{-\frac{t}{\tau}}$$

where $\beta$ is the phase shift, $\tau$ and $B$ are the decaying DC offset time constant and magnitude respectively, and $A_a$ and $A_b$ are the amplitudes of the sinusoidal component before and after the fault respectively.

Tests are designed to check the operation, which include testing over a range of frequencies from 47Hz to 53Hz and testing with different phase shift conditions.

A. Result Analysis

Figure 6 shows a comparison of the FCDFT and the phasor measurement by embedding in phase space (MEPS) with DC removed compared to the FCDFT of the original signal. The test data is generated from PSCAD with a fundamental frequency of 50 Hz and a sampling rate of 32 samples per cycle. It shows that both the FCDFT and MEPS with DC removed, do not have the large over and under shoot, which on the other hand exists in the FCDFT of the original signal. It also shows that the MEPS is significantly quicker than the FCDFT.

The performance is further analysed over the normal rated frequency range of frequencies, 47Hz to 53Hz for a protection relay. The phasor measurement results for 53Hz and 47Hz are shown in Fig. 7 and that for 50Hz was shown in Fig. 6. These figures show the method works correctly over this frequency range. In Fig. 7 the offset which would have otherwise been caused by the SE length not quite equalling half of the actual wavelength, has been compensated for and hence does not appear in the output. It also shows the oscillations, as in the FCDFT of the original signal, due to nominal “tuning” have been removed in the output by adaptive tuning.

The three cases of phase shift have been tested as shown in Fig. 8. Case 1 and case 2 can be illustrated using the same figure, as a case 1 phase shift is usually followed by...
Fig. 7. Comparison of the FCDFT and MEPS of the DC removed signal with relation to the FCDFT of the original signal for different frequencies

(a) Fundamental frequency = 53Hz
(b) Fundamental frequency = 47Hz

Fig. 8. Comparison of the FCDFT and MEPS of the DC removed signal with relation to the FCDFT of the original signal for the different phase shift conditions

(a) Case 3: Fault instant is at peak point
(b) Case 1/2: Fault instant is following/proceeding peak respectively

A case 2 phase shift. The figures show the measured current, determined DC offset, and current with dc removed, as well as the magnitude phasor measurement results. It can be seen that the correct value has been found for the DC offset, to successfully remove it from the signal. In the magnitude measurements, it can be seen that most of the large oscillations are removed. Though for case 1/2 the first oscillation is still significant, which may be due to the steep increase at the beginning of the fault.

B. Numerical Comparison

Table I presents the relative rise times of the different methods. The rise time is defined as the sample number when the signal reaches 90% of the fault settled value. The absolute number is irrelevant as the fault time is different in different cases. It allows several observations to be made: Firstly, the MEPS with DC removed (MEPS_r) is 0.5 to 0.63 cycles faster than the FCDFT with DC removed (FCDFT_r). Secondly, the delay between the MEPS_r and the FCDFT of original signal (FCDFT_o) is 0.11 cycles on average ranging between -0.03 cycles and 0.31 cycles. Table II shows the normalized maximum and minimum oscillation values respectively. The maximum and minimum oscillation values are the magnitude of the largest peak oscillation and largest trough oscillation respectively, normalized to the fault settled value. It can be seen that by removing the DC offset the overshoot is significantly reduced from an average of 8.4% to an average of 2.2% for MEPS_r and 1.0% for FCDFT_r. It can also be seen that the undershoot is significantly reduced from an average of 6.9% to an average of 0.7% for MEPS_r and 0.4% for FCDFT_r.

From these two tables it can be concluded that both MEPS_r and FCDFT_r provide a great improvement in accuracy, though coupled with the rise times, the advantage of the MEPS_r can be seen that it offers significantly shorter rise times.

IV. CONCLUSION

A practical method has been proposed to remove the DC offset in a half cycle plus an additional sample as a prepro-
TABLE I
THE RISE TIMES (SAMPLES)

<table>
<thead>
<tr>
<th>Rise Time</th>
<th>FCDFT,</th>
<th>MEPS,</th>
<th>FCDFT,</th>
</tr>
</thead>
<tbody>
<tr>
<td>50Hz</td>
<td>350</td>
<td>360</td>
<td>376</td>
</tr>
<tr>
<td>53Hz</td>
<td>347</td>
<td>353</td>
<td>372</td>
</tr>
<tr>
<td>47Hz</td>
<td>354</td>
<td>353</td>
<td>373</td>
</tr>
<tr>
<td>case1/2</td>
<td>249</td>
<td>249</td>
<td>266</td>
</tr>
<tr>
<td>case3</td>
<td>256</td>
<td>259</td>
<td>276</td>
</tr>
</tbody>
</table>

TABLE II
THE NORMALIZED MAXIMUM AND MINIMUM OSCILLATION VALUES FOR THE DIFFERENT METHODS.

<table>
<thead>
<tr>
<th>FCDFT,</th>
<th>MEPS,</th>
<th>FCDFT,</th>
<th>FCDFT,</th>
<th>MEPS,</th>
<th>FCDFT,</th>
</tr>
</thead>
<tbody>
<tr>
<td>50Hz</td>
<td>1.114</td>
<td>1.011</td>
<td>1.006</td>
<td>0.915</td>
<td>0.998</td>
</tr>
<tr>
<td>53Hz</td>
<td>1.066</td>
<td>1.013</td>
<td>1.009</td>
<td>0.905</td>
<td>0.992</td>
</tr>
<tr>
<td>47Hz</td>
<td>1.125</td>
<td>1.031</td>
<td>1.013</td>
<td>0.947</td>
<td>0.994</td>
</tr>
<tr>
<td>case1/2</td>
<td>1.060</td>
<td>1.048</td>
<td>1.019</td>
<td>0.953</td>
<td>0.991</td>
</tr>
<tr>
<td>case3</td>
<td>1.051</td>
<td>1.008</td>
<td>1.003</td>
<td>0.934</td>
<td>0.992</td>
</tr>
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REFERENCES


