Abstract—this paper presents the theory, design and implementation of a virtual instrument based on a new method to measure the power quality indices such as current harmonic distortion, voltage harmonic distortion and power factor as given by the IEEE standard. A modern spectral estimation technique (ESPRIT) is used to estimate the power quality indices. Simulation results followed by experimental results of the real time virtual instrument is presented to show the advantages and accuracy of using this method and further to show conformance of this device to well known standards such as IEEE and IEC.

Index Terms—power quality, power quality indices, spectral analysis, windowed ESPRIT.

I. INTRODUCTION

Power quality is an emerging issue in electricity distribution networks due to more nonlinear loads being added to the power grid[1]. Electronic equipment and nonlinear loads produce various events in the power system like harmonics in the voltage and current signals. Sags or swells are caused by power system faults and switching events of capacitor banks. Moreover, phase and magnitude imbalances can occur due to single-phase conditioning or load unbalance[1]. IEEE Std 1459-2010 [2] gives definitions and equations for calculating various power quality indices. Further standards such as IEEE Std 1159[3], IEC Standard 61000-4-7[4] and 61000-4-30[5] provide benchmarks for the testing of equipment and further technical information on how to calculate power quality indices.

Many signal processing methods have been examined in the literature for finding power quality indices, examples include the Fourier Analysis in [6][7][8], S-transform[9][10] and the wavelet family of transform [11][12]. S-transform is computationally expensive compared to the Fourier analysis techniques and further inherits the same disadvantage of leakage associated with the windowing techniques. The wavelet family of transform such as the Discrete Wavelet Transform (DWT) are not suitable for harmonic detection as once the sampling frequency is set the center frequency for dyadic filters cannot be changed and may not coincide with the harmonic frequencies[13][14].

There have been recent publications on the use of modern spectral estimation techniques for detecting power quality events such as estimating the harmonics and inter-harmonics[15][16] and also the measurement of the respective amplitudes as in [16][17], however none of these publications have calculated the power quality indices as outlined in the IEEE std 1459-2010. This paper provides a new approach to the calculation of the power quality indices as outlined in the IEEE 1459-2010 standard. Particularly a unified method based on subspace techniques is presented for calculating all the power quality indices as outlined in IEEE 1459-2010. Moreover some extra insight and suggestions are made to further optimize the method for spectral estimation.

The paper is organized as follows. Section II briefly presents the method of ESPRIT. Section III describes various power quality indices defined in IEEE Std 1459-2010. The design of the virtual instrument is presented in Section IV. Section V and VI present the simulation and experimental results for our virtual instruments. Section VII concludes the paper.

II. ESPRIT

Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) is a modern method of spectral estimation which was introduced by Roy and Kailath in [18] and more explanation was given in[15][19][20][21]. This method exploits an invariance property that naturally exists in discrete periodic signals.

The ESPRIT algorithm requires the underlying process to be a band limited sum of sinusoidal signals; the signals can be either complex or real in nature, this makes the method particularly suitable for our application of power quality analysis. There are two approaches to ESPRIT; by working only on the time data [20] and by working on the estimated auto-correlation matrix [17]. In our analysis we will be working on the estimated auto-correlation matrix [22]. Although the autocorrelation method increases the computational cost of the system, but it also behaves as a filter further averaging the noise in our signal and providing a better estimate for the frequencies and subsequently the amplitudes and phase angles.

ESPRIT is suited for periodic complex signals with additive noise modeled as:

\[ y[n] = \sum_{i=1}^{K} A_i e^{j\Omega_i n} + w[n] \]  \hspace{1cm} (1)

Where frequencies \( \Omega_i \) are distinct and satisfy \(-\pi < \Omega_i < \pi\), and

\[ E\{w[k]\} = 0 \]  \hspace{1cm} (2)
If we define an \( m \times 1 \) vector signal \( y[n] \) as
\[
y[n] = [v[n] \ v[n+1] \ldots \ v[n+m-1]]^T,
\]
then it can be expressed in terms of its components as follows:
\[
y[n] = \begin{bmatrix}
e^{\lambda_1} & e^{\lambda_2} & \cdots & e^{\lambda_K} \\
e^{\lambda_1(n-1)} & e^{\lambda_1(n-2)} & \cdots & e^{\lambda_1(n-K)} \\
e^{\lambda_2(n-1)} & e^{\lambda_2(n-2)} & \cdots & e^{\lambda_2(n-K)} \\
\vdots & \vdots & \ddots & \vdots \\
e^{\lambda_K(n-1)} & e^{\lambda_K(n-2)} & \cdots & e^{\lambda_K(n-K)}
\end{bmatrix}_{n=1}^{K} \begin{bmatrix} A_1e^{\lambda_1} \\ A_2e^{\lambda_2} \\ \vdots \\ A_Ke^{\lambda_K}
\end{bmatrix}
\]
\[
+[w[n] \ w[n+1] \ldots \ w[n+m-2] \ w[n+m-1]]^T
\]
The (time averaged) auto correlation of a vector signal (suitable for working with periodic signals corrupted by noise) is defined as follows:
\[
R_{yy}[n] = \lim_{N \to \infty} \frac{1}{N} \sum_{k=0}^{N-1} E\{y[k]y^H[k-n]\}.
\]
It gives a measure of similarity between \( y[n] \) and the delayed version of \( \overline{y} \).

The whole ensemble. For a stationary random processes or a wide sense stationary random process, \( E\{y[k]\y[k-n]\} \) just depends on \( n \) and the time averaging in (4) is unnecessary. So, this definition is a generalization of the more usual definition where no time averaging is performed. Also, this definition reduces to the usual definition for the deterministic periodic signals where ensemble consists of just one number, i.e. \( E\{y\} \) in (4) is unnecessary.

It follows that
\[
R_{yy}[0] = V V^H + \sigma_w^2 I_m + \sigma_v^2 I_m
\]

Note that we will have \( m > K \) and \( V \) is a Vandermonde matrix having rank \( K \) (since frequencies \( \Omega_k \) are distinct and satisfy \( -\pi < \Omega_k < \pi \)), i.e. columns of \( V \) are linearly independent and its range space is a \( K \)-dimensional subspace of \( \mathbb{C}^m \). Key to determining unknown parameters is to work with the range space of \( V \). By its construction \( R_{yy}[0] \) is positive semidefinite matrix (positive definite for \( \sigma_w^2 > 0 \)).

In fact any vector \( x \) (of size \( m \times 1 \)) in the null space of \( V^H \) (i.e. \( V^H x = 0 \)) is an eigenvector of \( R_{yy}[0] \) with corresponding eigenvalue \( \sigma_w^2 \), this is because:
\[
R_{yy}[0] x = V D V^H x + \sigma_w^2 I_m x = \sigma_w^2 x
\]

Since the dimension of the null space of \( V^H \) is \( m-K \), \( m-K \) of the eigen values of \( R_{yy}[0] \) are \( \sigma_w^2 \) having \( m-K \) linearly independent eigenvectors. Being a Hermitian matrix, \( R_{yy}[0] \) permits the following eigen decomposition
\[
R_{yy}[0] = U \Lambda U^H
\]

Without loss of generality, we can assume that the eigen values are arranged in decreasing order. Then the last \( m-K \) of them would be \( \sigma_w^2 \) (a fact that can be used to determine \( K \) and \( \sigma_w^2 \)) and the corresponding eigenvectors will span the null space of \( V^H \) based on above discussion. Since the orthogonal complement of the null space of \( V^H \) is the range space of \( V \), we have that the first \( K \) eigenvectors span the range space of \( V \). Thus, we can partition \( U \) as follows
\[
U = [U_{s_1} \ U_{s_2}]
\]

Where \( V^H U_w = 0 \) and we can find a \( K \times K \) nonsingular matrix \( T \) such that
\[
V = U_T
\]

This fact now can be used to evaluate the unknown frequencies \( \Omega_k \) as follows. Consider the sub matrices of \( V \) formed by the first \( (m-1) \) rows and last \( (m-1) \) rows, i.e. \( V_1 \) and \( V_2 \) respectively as shown in (5). Denoting the corresponding sub matrices of \( U \) formed by the first \( (m-1) \) rows and last \( (m-1) \) rows of \( U_{s_1} \) and \( U_{s_2} \) respectively, from (11) we have
\[
V_1 = U_{s_1} T \quad \text{and} \quad V_2 = U_{s_2} T
\]

Due to structure of \( V \) from (5), it can be seen that \( V_1 \) and \( V_2 \) are related as follows:
\[
V_2 = V_1 \Phi
\]

where \( \Phi \) is defined as:
\[
\Phi = \begin{bmatrix}
e^{\Omega_1} & 0 & \cdots & 0 \\
0 & e^{\Omega_2} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & e^{\Omega_K}
\end{bmatrix}
\]

Then from (12) and (13) we get that
\[
U_{s_1} \Phi = U_{s_2} \Rightarrow \Phi = T^{-1} \left( U_{s_1}^m U_{s_2}^{-1} U_{s_1} U_{s_2}^h \right) T
\]

From above it can be seen that the \( K\times K \) matrix \( \Phi \) is related to the matrix \( (U_{s_1}^m U_{s_2}^{-1}) U_{s_1} U_{s_2}^h \) through a similarity transform and thus both should have the same eigenvalues. But the eigenvalues of \( \Phi \) are easily seen to be \( e^{i\lambda_i} \). So if the eigenvalues of \( (U_{s_1}^m U_{s_2}^{-1}) U_{s_1} U_{s_2}^h \) are denoted by \( \lambda_i \) (or equivalently \( |\lambda_i| e^{i\theta_i} \) in the polar coordinates), then the unknown frequencies can be computed as follows
\[
\Omega_k = \angle \lambda_i, \quad i = 1, \ldots, K.
\]

The magnitude of eigenvalues should theoretically be 1, but would vary slightly in the implementation due to numerical precision and finite window length.
Remark: Computation of \((U_H^H U_s)\)^{-1} can be done efficiently without doing an explicit matrix inversion by using matrix inversion lemma. Noting that \(U\) is a unitary matrix, we must have \(U_H^H U = I_n\), from which it easily follows that \(U_H^H U_s = I_K\).

If the last row of \(U_s\) were denoted by \(u_m\), then we have

\[
I_K = U_H^H U_s = \begin{bmatrix} U_H^H & u_m \end{bmatrix} \begin{bmatrix} U_s \end{bmatrix}^H = U_H^H U_s + u_m u_m^H
\]

\[
\Rightarrow (U_H^H U_s)^{-1} = (I_K + u_m (-1)^{-1}) u_m^H
\]

\[
= I_K - I_K u_m u_m^H + (-1)^{-1} u_m^H I_K
\]

Once the frequencies are determined, then amplitudes can be obtained from (7) as follows

\[
D = (V^H V)^{-1} V^H R_{yy}^0 V (V^H V)^{-1} - \sigma^2_n (V^H V)^{-1}
\]

The phase (as well as magnitude) information can be obtained from least square estimate \(\hat{s}[n]\) of \(s[n]\) from (5) as follows

\[
\hat{s}[n] = (V^H V)^{-1} V^H y[n]
\]

\[
\Rightarrow \begin{bmatrix} \hat{A}_1 & \hat{A}_2 & \ldots & \hat{A}_K \end{bmatrix}^T
\]

\[
= (\Phi^*)^H (V^H V)^{-1} V^H y[n]
\]

Although the above scheme is able to compute the unknown parameters, it cannot be implemented exactly since the computation of \(R_{yy}^0\) requires us to observe the signal over an infinite interval. However it is possible to compute an approximation \(\hat{R}_{yy}\) of \(R_{yy}^0\) by observing the signal over a finite window of length \(M\) as follows:

\[
X = \begin{bmatrix} v[0] & v[1] & \cdots & v[M-m] \\
\vdots & \vdots & \ddots & \vdots \\
v[m-1] & v[m] & \cdots & v[M-1] \end{bmatrix}
\]

\[
\hat{R}_{yy} = \frac{1}{(M-m)} XX^H
\]

This estimation basically amounts to truncating the infinite summation in (6) to a finite length \(N = M - m\). Obviously the estimates get better with larger \(M\) (but will result in poor time resolution). In our scheme, we follow the above procedure by replacing \(R_{yy}^0\) by \(\hat{R}_{yy}\) i.e. by doing the eigenvalue decomposition of \(\hat{R}_{yy}\). Estimation of \(\Phi\) in (14) then amounts to obtaining the least square estimate, assuming that \(\hat{U}_{s2}\) is an exact estimate of \(U_{s2}\) (which is clearly incorrect). This can be improved by using the total least squares solution at the expense of an increased computational complexity. The computations are repeated over successive non-overlapping windows of length \(M\). Moreover the length \(M\) is important when trying to distinguish between closely spaced frequencies. The autocorrelation defined in (6) contains complex terms, some of them with typically low frequencies. These frequencies are equal to the difference in the frequencies of adjacent frequency components \((f_{\text{difference}})\) in the observed signals frequency spectrum. These terms average to zero if the data length is very long or if the autocorrelation is performed over their period (the smaller the frequency difference the bigger the period required). Since observing the data for a long period is not possible and further not desirable, we are required to observe the data at least for a period of \((1/f_{\text{difference}})\).

Finally the typical voltages and currents to be estimated in the power system would be of the form

\[
v_i(t) = \sum_{k=1}^{\infty} \sqrt{2} V_{\text{sh}} \sin(\omega_k t + \theta_{\text{sh}}) + w(t)
\]

where \(\omega_{\text{sh}} = h \omega_1\) with \(\omega_1\) being the fundamental frequency. The ESPRIT method does not assume that the underlying frequencies in the signal are related as in (21) and is suitable for detecting inter-harmonics as well. They estimate a finite number of frequencies where the signal power is more than the noise power. The method works with the samples of the signals and each sinusoid in (21) is represented by two complex exponentials

\[
\left( \frac{V_{\text{sh}}}{\sqrt{2}} e^{j(\theta_{\text{sh}} - (\pi/2))} \right) e^{j(2\pi\omega_k t + \theta_{\text{sh}})}
\]

and

\[
\left( \frac{V_{\text{sh}}}{\sqrt{2}} e^{-j(\theta_{\text{sh}} - (\pi/2))} \right) e^{-j(2\pi\omega_k t + \theta_{\text{sh}})}
\]

where \(\omega_{\text{sh}}\) is the sampling frequency in rad/sec, i.e. for each sinusoid we get two frequencies of \(\pm \frac{2\pi\omega_{\text{sh}}}{\omega_k}\) and the corresponding amplitudes are complex conjugates of each other.

### III. POWER QUALITY INDICES

For efficient transfer of power between the generator and the customer, the power system should be balanced and operate with a unity power factor. However, all practical power systems deviate from this ideal outcome. The IEEE Std 1459-2010 [2] defines the power quality indices to quantify these deviations. It handles single-phase and three-phase power systems under sinusoidal, non-sinusoidal, balanced or unbalanced conditions. This standard and a closely related documents [23][24] separate the fundamental and non-fundamental components in power. Furthermore, they separate the active and non-active components for power quality assessment. We examine a three-phase, four-wire system operating under non-sinusoidal and unbalanced conditions that can be looked as the most general case. All the definitions that have been used in our simulation and experiment are presented here [2][23].

Since the signals in the power system are periodic, we would make an extensive use of Fourier series and phasor notation in the following development. These components can
also be estimated using the ESPRIT technique as discussed in the previous section.

Time signals will usually be denoted by the lowercase symbols. For example, voltage in phase $a$ (which is periodic with a period $2\pi/\omega_a$, sec) will have a Fourier series expansion

$$v_a(t) = \sum_{h=1}^{\infty} \sqrt{2} V_{ah} \sin(h \omega_a t + \phi_{ah}).$$

(22)

where, $V_{ah}$ denotes the rms value and $\phi_{ah}$ denotes the phase angle of the $h^{th}$ harmonic of voltage in phase $a$. For clarity purposes, whenever convenient, we will use bold uppercase letters to denote these harmonics as phasors, e.g. $V_{ah} = V_{ah} e^{j\phi_{ah}}$ denotes the phasor corresponding to the $h^{th}$ harmonic with the frequency $h\omega_a$ understood implicitly. Similarly the phase current for phase $a$ will have a Fourier series expansion

$$i_a(t) = \sum_{h=1}^{\infty} \sqrt{2} I_{ah} \sin(h \omega_a t + \phi_{ah}).$$

(23)

$v_{ab}(t)$ will denote the line-to-line voltage between phase $a$ and phase $b$ and $i_n(t)$ will denote the neutral current. Other phase voltages, line-to-line voltages and currents are defined in a similar fashion. With these notations we are ready to define various power quality indices and involved quantities.

The concept of effective line current ($I_e$) is based on the line current of a virtual balanced circuit having the same losses as the actual unbalanced circuit and is given by

$$I_e = \left( I_{e1}^2 + I_{e2}^2 + I_{e3}^2 \right)^{\frac{1}{2}}$$

(24)

which can be re-expressed as

$$I_e^2 = \sum_{h=1}^{\infty} I_{eh}^2.$$ 

(25)

As (25) includes the effect of all the harmonics, the fundamental component is separated as the effective fundamental line current ($I_{e1}$) given by

$$I_{e1} = \left( I_{e1}^2 + I_{e2}^2 + I_{e3}^2 \right)^{\frac{1}{2}}.$$ 

(26)

and all the other harmonics are lumped together as effective non-fundamental line current ($I_{eH}$) given by

$$I_{eH} = (I_e^2 - I_{e1}^2).$$

(27)

While the fundamental component of the current is involved in the effective transfer of power, the harmonics lead to increased losses in the system and are thus undesirable. This undesirable feature is quantified by total harmonic distortion of the line current ($THD_e$).

$$THD_e = (I_{eH} / I_e).$$

(28)

The effective line-to-neutral voltage ($V_e$) is given by

$$V_e = \frac{(V_{e1}^2 + V_{e2}^2 + V_{e3}^2)}{6} + \frac{(V_{e1}^2 + V_{e2}^2 + V_{e3}^2)}{18}.$$ 

(29)

Based on the result that the phasor ($V_{ah} - V_{bh}$) has a length given by $\sqrt{V_{ah}^2 + V_{bh}^2 - 2V_{ah}V_{bh}\cos(\theta_{ah} - \theta_{bh})}$, (29) can be re-expressed as

$$V_e^2 = \frac{\sum_{h=1}^{\infty} (V_{ah}^2 + V_{bh}^2 + V_{eh}^2)}{6} + \frac{\sum_{h=1}^{\infty} (V_{ah}^2 + V_{bh}^2 - 2V_{ah}V_{bh}\cos(\theta_{ah} - \theta_{bh}))}{18}.$$ 

(30)

Working along similar lines, the effective fundamental line-to-neutral voltage ($V_{e1}$) is given by

$$V_{e1} = \frac{(V_{e1}^2 + V_{e2}^2 + V_{e3}^2)}{6} + \frac{(V_{e1}^2 + V_{e2}^2 + V_{e3}^2)}{18}.$$ 

(31)

and can be re-expressed as

$$V_{e1}^2 = \frac{(V_{e1}^2 + V_{e2}^2 + V_{e3}^2)}{6} + \frac{(V_{e1}^2 + V_{e2}^2 - 2V_{ah}V_{bh}\cos(\theta_{ah} - \theta_{bh}))}{18}.$$ 

(32)

effective non-fundamental line-to-neutral voltage ($V_{eH}$) is given by

$$V_{eH} = (V_e^2 - V_{e1}^2).$$

(33)

and the total harmonic distortion of the line-to-neutral voltage ($THD_e$) is defined as

$$THD_e = (V_{eH} / V_e).$$

(34)

The effective apparent power ($S_e$) is given by

$$S_e = 3V_eI_e$$

(35)

which can again be resolved into the component due to the fundamental, known as the fundamental effective apparent power ($S_{e1}$).

$$S_{e1} = 3V_{e1}I_{e1}$$

(36)

and the other harmonics as non-fundamental effective apparent power ($S_{eH}$)

$$S_{eH} = (S_e^2 - S_{e1}^2)$$

(37)

Using (27)(33), $S_{eH}$ can be further decomposed as

$$S_{eH} = D_{eH}^1 + D_{eH}^3 + S_{eH}^\prime,$$

(38)

where $D_{eH}$ is the current distortion power

$$D_{eH} = 3V_{eH}I_{eH},$$

(39)

$D_{eH}$ is the voltage distortion power

$$D_{eH} = 3V_{eH}I_{eH},$$

(40)

and $S_{eH}$ is the harmonic apparent power

$$S_{eH} = 3V_{eH}I_{eH}.$$ 

(41)

The active power ($P$) quantifies the average power delivered to the load, and is given by
\[
P = \left( \sum_{k=1}^{n} V_{ab} I_{ab} \cos(\theta_{ab} - \phi_{ab}) \right) + \sum_{k=1}^{n} V_{bb} I_{bb} \cos(\theta_{bb} - \phi_{bb}) + \sum_{k=1}^{n} V_{cb} I_{cb} \cos(\theta_{cb} - \phi_{cb}) \right) .
\]

The line utilization is quantified by power factor \( P_f \)

\[
P_f = \left( P / S_e \right).
\]

Note that active power \( (P) \) includes the power transferred by the non-fundamental components. However, harmonics are undesired in a power system and most efficient transfer of power requires that there be no voltage and current harmonics, line voltages and currents have same rms values (i.e., \( V_{a1} = V_{b1} = V_{c1} , I_{a1} = I_{b1} = I_{c1} \)), line currents be in phase with the line voltages (i.e. \( \phi_{a1} = \theta_{a1} , \phi_{b1} = \theta_{b1} , \phi_{c1} = \theta_{c1} \)), and the phase of line voltages are related as \( \theta_{b1} = \theta_{a1} - (2\pi / 3) \), \( \theta_{c1} = \theta_{a1} - (4\pi / 3) \). Symmetrical components are used to measure deviations from these conditions, by transforming the voltage and current phasors into positive \( (V_{h1} = V_{b1} e^{j\theta_1}) \), negative \( (V_{h1} = V_{c1} e^{j\theta_1}) \), and zero-sequence \( (V_{h1} = V_{a1} e^{j\theta_1}) \) components as follows:

\[
\begin{bmatrix}
V_{h1}' \\
V_{h1}' \\
V_{h1}' \\
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
e^{j2\pi/3} & e^{j4\pi/3} & 1 \\
e^{j4\pi/3} & e^{j2\pi/3} & 1 \\
1 & 1 & 1
\end{bmatrix} \begin{bmatrix}
V_{ab} \\
V_{bb} \\
V_{cb}
\end{bmatrix}.
\]

For the ideal scenario, the non-fundamental components are absent and we have \( V_{h1}' = V_{a1} , V_{h1}' = 0 , V_{h1}' = 0 \). The usual desirable component of the average power delivered to the load is quantified by the fundamental positive sequence active power \( (P_f^+ ) \)

\[
P_f^+ = 3V_{h1}' I_{h1}' \cos(\theta_{h1}' - \phi_{h1}'),
\]

where we have denoted the symmetric components of the current phasors in a similar fashion as \( I_{h1}' = I_{h1} e^{j\phi_1} \). Line utilization for the desirable fundamental component is given by the positive sequence power factor \( (P_f^+) \)

\[
P_f^{+} = \cos(\theta_{h1}' - \phi_{h1}').
\]

This can also be expressed in the usual manner as \( P_f^{+} = (P_f^+ / S_e^+) \), where \( S_e^+ = 3V_{h1}' I_{h1}' \) is the fundamental positive sequence apparent power.

So our procedure is to acquire one frame of voltage and current measurements, estimate various magnitudes and respective phase angles and use them in turn to estimate the power quality indices. This needs to be continued with the next frame of measurements. Even for stationary signals, our estimates will show some fluctuation due to underlying noise in the signals, presence of inter-harmonics, and the alignment of the frames with the signals.

IV. VIRTUAL INSTRUMENT DESIGN

This section presents the design for the virtual instrument/meter. The system is designed for a three-phase system, although it can easily be used/changed for a single-phase system. Fig. 1 presents the salient features of the design for the virtual instrument using the ESPRIT method of spectral estimation.

In order to use the ESPRIT method in an instrument for detecting PQ indices a few steps need to be taken. As discussed in section III, the ESPRIT method assumes a certain model for the underlying process (i.e. the number of complex signals to be known). The number of harmonics is important for the optimal performance of the system when deciding about the correlation matrix size \( \hat{R}_{\omega} \). The auto-correlation matrix size is an important factor when dealing with real-time instruments as the eigenvalue decomposition is an expensive task of \( O(m^3) \). This said prefiltering is an important step when using ESPRIT as it limits the number of harmonics and inter-harmonics present.

Various methods in the literature have been proposed to estimate the number of harmonics present and we direct readers to those publications [21]. In our design for the virtual instrument we assume only harmonics present, i.e. no interharmonics. Also as it can be seen from the diagram simple calculations have to be performed in order to get the phase difference between the current phases, voltage phases and current-voltage phases. The method can be summarized as follows, (i) rms values and phase angles of voltage and current harmonics can be estimated from the ESPRIT algorithm (ii) phase estimates from (i) can be used to obtain various phase angle differences such as \( (\theta_{ab} - \theta_{cb} ) , (\theta_{ab} - \theta_{ca} ) , (\theta_{cb} - \theta_{ca} ) \), \( (\phi_{ah} - \phi_{bh} ) , (\phi_{ah} - \phi_{ch} ) , (\phi_{bh} - \phi_{ch} ) \) and \( (\theta_{bh} - \theta_{ch} ) , (\theta_{ab} - \theta_{bc} ) ). Finally, power quality indices are computed from these estimates as detailed in section III.

IEC61000-4-7, IEC61000-4-30 standard describes the testing and measurement techniques for harmonics and inter-harmonics. Further it specifies measurement criteria for power quality instrumentation. In this document we do not compute the indices specified in the IEC standards however our virtual instrument design conforms to these standards, by using a window length of no more than 0.2 seconds to estimate the power quality indices. Further IEEE Std 1159-2009 specifies many sets of power quality events. In general, these events can be classified as short duration events (e.g. sags and swells), long duration events (e.g. sustained interruption), imbalance, and waveform distortions (e.g. harmonics) etc. Our virtual instrument can detect both short and long duration events.

V. SIMULATION

The virtual instrument described in the previous section was implemented in Simulink [25] under MATLAB and simulated expensively. The purpose of these simulations was to validate the design and test them in the presence of measurement noise (which is harder to achieve in experiments). The Simulink design permits an easier real time implementation on a DSpace board [26]. A sampling frequency of 1 kHz was used.
A. Simulation 1: High Measurement Noise

A three-phase four-wire power system operating under unbalanced and non-sinusoidal conditions was simulated. The signals contained second harmonic (100 Hz) and fourth harmonic (200 Hz). The presence of second harmonic was selected to test the frequency resolution of the windows as it is only 50 Hz away from the fundamental. The harmonic levels chosen for the simulation conform to IEEE Std 1159-2009 [3] that specifies steady state harmonic levels between 0% and 20%. The standard also specifies that noise in the system can reach up to 1% of the fundamental. The specification for the voltage and current signals used in the simulation is stated in Table I. Independent additive Gaussian white noise with a variance of 1.8 was added to each of the phase voltages (line currents).

Two sets of simulation was run for several seconds (i) with a buffer size of 80 samples and (ii) a buffer size of 120 samples. All the power quality indices mentioned in section III were computed. However, representative graphs for only three of them (\(P_f\), \(THD_i\), and \(THD_v\)) have been plotted here due to space constraints. Fig. 2 shows the percentage error (i.e. absolute error in our estimates as a percentage of the nominal values) for the ESPRIT method. Further Table II below summarizes the results for the rest of the power quality indices both for the 80 point and 120 point buffers. From the table it can be seen that the percentage error difference between the two buffers is much smaller than 1%.

![Diagram of ESPRIT method](image)

**Fig. 1. Virtual instrument design using ESPRIT method.**

![Graph showing percentage error](image)

**Fig. 2. Percent error in \(P_f\), \(THD_i\), \(THD_v\) for simulation 1 using ESPRIT virtual instrument with an 80 point buffer.**

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>(h = 1)</th>
<th>(h = 2)</th>
<th>(h = 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V_{a}) (Volts RMS)</td>
<td>240</td>
<td>24</td>
<td>2.4</td>
</tr>
<tr>
<td>(V_{b})</td>
<td>242</td>
<td>24</td>
<td>2.4</td>
</tr>
<tr>
<td>(V_{c})</td>
<td>235</td>
<td>24</td>
<td>2.4</td>
</tr>
<tr>
<td>(I_{a})</td>
<td>1</td>
<td>0.1</td>
<td>0.01</td>
</tr>
<tr>
<td>(I_{b})</td>
<td>0.8</td>
<td>0.12</td>
<td>0.011</td>
</tr>
<tr>
<td>(I_{c})</td>
<td>0.9</td>
<td>0.11</td>
<td>0.012</td>
</tr>
<tr>
<td>(I_{ab})</td>
<td>0.2341</td>
<td>0.1509</td>
<td>0.0121</td>
</tr>
<tr>
<td>(\theta_{a}) (Degrees)</td>
<td>3°</td>
<td>30°</td>
<td>145°</td>
</tr>
<tr>
<td>(\theta_{b})</td>
<td>245°</td>
<td>210°</td>
<td>165°</td>
</tr>
<tr>
<td>(\theta_{c})</td>
<td>120°</td>
<td>150°</td>
<td>200°</td>
</tr>
<tr>
<td>(\phi_{ab})</td>
<td>5°</td>
<td>20°</td>
<td>38°</td>
</tr>
<tr>
<td>(\phi_{bc})</td>
<td>242°</td>
<td>210°</td>
<td>165°</td>
</tr>
<tr>
<td>(\phi_{ca})</td>
<td>120°</td>
<td>0°</td>
<td>200°</td>
</tr>
</tbody>
</table>
TABLE II
PERCENT ERROR FOR SIMULATION 1 USING A 120 POINT BUFFER (120p) AND AN 80 POINT BUFFER (80p).

<table>
<thead>
<tr>
<th>PQ Indices</th>
<th>% error mean 120p</th>
<th>variance 120p</th>
<th>% error mean 80p</th>
<th>variance 80p</th>
</tr>
</thead>
<tbody>
<tr>
<td>$PF_1^+$</td>
<td>0.126</td>
<td>0.003</td>
<td>0.200</td>
<td>0.011</td>
</tr>
<tr>
<td>$P_1^*$</td>
<td>0.100</td>
<td>0.002</td>
<td>0.176</td>
<td>0.009</td>
</tr>
<tr>
<td>$S_{Hf}$</td>
<td>0.644</td>
<td>0.108</td>
<td>0.616</td>
<td>0.1193</td>
</tr>
<tr>
<td>$S_1$</td>
<td>0.005</td>
<td>0.000</td>
<td>0.008</td>
<td>0.000</td>
</tr>
<tr>
<td>$V_e$</td>
<td>0.036</td>
<td>0.000</td>
<td>0.060</td>
<td>0.001</td>
</tr>
<tr>
<td>$I_e$</td>
<td>0.016</td>
<td>0.000</td>
<td>0.017</td>
<td>0.000</td>
</tr>
</tbody>
</table>

B. Simulation 2: Rectified RL Circuit

For this simulation the load consisted of a half-wave rectifier in each phase supplying power to an unbalanced RL circuit as shown in Fig. 3. The voltages across the different resistors were treated as phase voltages to create unbalance. The phase voltages and line currents had harmonics due to the rectifier. In this case the buffer size for ESPRIT method is set to 80(4 cycles) samples.

VI. REAL TIME IMPLEMENTATION AND EXPERIMENT

In order to verify the simulations, the virtual instrument designed in section V was implemented in real time using a DSpace board. The rectified RL circuit of simulation 2 was also setup and it was used to test the real time instrument. A three-phase power supply with a line-to-line voltage of 415 Volts was used to supply the power. The sampling frequency of 1 kHz was used, which is same as the one used for the above simulations. Again, all the power quality indices in section III were computed Table III states the percentage error and variance for the PQ indices.

TABLE III
PERCENT ERROR FOR EXPERIMENT 1 USING AN 80 POINT BUFFER (80p).

<table>
<thead>
<tr>
<th></th>
<th>$PF_1^+$</th>
<th>$P_1^*$</th>
<th>$S_{Hf}$</th>
<th>$S_1$</th>
<th>$V_e$</th>
<th>$I_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>% error mean</td>
<td>0.107</td>
<td>1.285</td>
<td>1.96</td>
<td>0.382</td>
<td>0.896</td>
<td>1.151</td>
</tr>
<tr>
<td>variance</td>
<td>0.003</td>
<td>0.483</td>
<td>2.386</td>
<td>0.035</td>
<td>0.896</td>
<td>0.05321</td>
</tr>
</tbody>
</table>

Fig. 5 shows a representative graph for the percent error in estimating power factor ($P_f$) for the real time implementations of the virtual instrument using the ESPRIT method.

VII. CONCLUSION

A virtual instrument to compute power quality indices based on ESPRIT was presented. The ESPRIT method was found to be accurate with percentage error of less than 5%. It could also detect all the important power quality indices within 4 cycles or less of the fundamental harmonic; this
makes the ESPRIT method very competitive with respect to other DSP methods such as the Fourier and windowed methods of analysis which typically suffer from artifacts of windowing. The downside to the ESPRIT method is calculating the Singular Value Decomposition of the autocorrelation matrix which makes it computationally more expensive compared to fast Fourier based methods.

VIII. REFERENCES


IX. BIOGRAPHIES

Reza Zolfaghari received his bachelor's degree in computer engineering from the University of New South Wales, Sydney, Australia in 2003. He is currently completing his Master of Philosophy degree in Electrical Engineering from the University of Sydney, Sydney, Australia. His main area of research currently is Digital Signal Processing and Spectral Estimation techniques applied to Power quality.

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