Simulation study of a "Smart Grid" approach: model reduction, reactive power control

Bernhard Wille-Haussmann, Jochen Link, and Christof Wittwer

Abstract—Distribution grids evolve more and more to so called “Smart Grids” with a decentralized structure. New information and communication technologies (ICT) to manage these “Smart Grids” are necessary to integrate a high share of renewable energy systems. Due to a high number of system components the optimal operation and control of “Smart Grids” is highly complex which is why this paper uses the approach of symbolic reduction to derive models with lower order for grid segments. Symbolic model reduction is a technique that analyses a set of equations and removes equations that do not influence the specified variables. The simulation study on a typical distribution grid shows a reduction by the factor of 2. The method of symbolic reduction has been applied to several sub grids of the CIGRE benchmark grid - a typical distribution grid. Local control strategies for feeding reactive power have been implemented for 10 cogeneration plants. Results show that the spread and the absolute value of the voltage band in the CIGRE grid decrease. This offers further degrees of freedom for the grid operation and further distributed generators can be connected.

Index Terms—distributed generation, grid control, reactive power.

I. INTRODUCTION

WITHIN the Strategic Energy Technology Plan (SET-Plan) [12] the European Commission defines concrete targets for climate and energy policy proclaiming the so called 20-20-20 target. Greenhouse gas emissions and primary energy usage should be reduced until 2020 by 20 % each. Simultaneously, the fraction of renewables of the total energy consumption should be increased by 20 %. Big offshore wind parks are supposed to be the most dominant renewable energy source.

In Germany targets defined within the SET-Plan are implemented by the lead study of the German Federal Ministry for the Environment, Nature Conservation and Nuclear Safety (BMU) [5]. In 2009 the study has been updated and predicts renewable energy resources with an installed power of 79 GW in 2020. All these generators will produce 196 TWh which corresponds to 35 % of the predicted electricity demand. In 2050 an installed capacity of 130 GW renewable power sources should generate about 80 % of the total German electricity demand.

Fluctuating generation influences the residual grid load\(^1\) and with it the operation of conventional power plants. Fig. 1 shows the duration curve of Germany’s vertical grid load in 2008 [10]. By comparing the load curve with installed capacity of renewable generators it becomes clear that in 2008 the capacity is in the range of the base load. As a result a shutdown of base load generation will typically not be necessary. Furthermore, targets of the BMU lead scenario [5] are above the peak load. In times with a generation surplus this will lead to negative residual grid loads and no conventional generation can be feed into the grids. In times with smaller renewable potential it will be necessary to cover load out of other controllable generation units, storages, or import of electricity to get rid of the high variability in generation.

In order to achieve a balance between generation and load in the future it will not be sufficient to just control generation. Thus, with a rising share of fluctuating generation it will also be necessary to have loads matching generation. In the study “Smart Distribution 2020” [4, the Association for Electrical, Electronic and Information Technologies (VDE) suggests the usage of time variable electricity tariffs to control generators. This approach has been picked up in [16] where variable local feed-in tariffs are used to transport global grid restrictions to the local plant operator. The local plant uses this as a “soft” control signal. For example, [17] uses variable tariffs to calculate day ahead optimized schedules for cogeneration plants (CHP). Thereby, a model based on mixed linear programming is used to describe the system consisting of CHP, auxiliary boiler, and heat storage.

Compared to the day-ahead scheduling with a lot of uncertainties controlling the actual operation point of the local system is much more time critical. Compared to thermal systems electrical systems have very small time constants. If a
II. MODEL REDUCTION OF ELECTRICITY GRIDS

Usually, distribution grids in “Smart Grids” consist of a great number of consumers and generators which are interconnected by lines and transformers. With the well known equivalent circuits for grid elements and loadflow analysis methods, e.g. described in [8], it is possible to model arbitrary grids. For getting the loadflow of a symmetric balanced electricity grid with \( n \) nodes the loadflow equations

\[
S_i^+ = P_i - jQ_i = \sum_k (Y_{ik} V_k), \quad \forall i \in 1, \ldots, n
\]

must be solved to get the node voltages \( V_i \) and currents on the lines. \( Y \) is the \( n \times n \) admittance matrix and describes the grid structure completely. However, for analyzing control routines or in order to optimize the system this complete model is much too detailed. For optimization either single grid segments can be regarded or adequate model reduction techniques must be used.

In literature various techniques for simplifying the loadflow equations (1) have been discussed. The method of DC-loadflow calculation linearizes the loadflow equations, neglects line resistances and assumes that voltages vary marginally in amplitude and phase. In [19] this method is discussed in order to describe active power flow. An error smaller than 5% can be guaranteed only if ratio of impedances \( \frac{\Delta V}{R} \geq 4 \), i.e. \( R << X \). In distribution grids this requirements can not be satisfied.

Akhavein [18] compares reduced models for analysis of outage risk and concentrates on simplifying the surrounding of the grid to be examined. One disadvantage of these methods is that only one static load point with different outage scenerios is regarded. Tekiner [21] combines these models with a Monte-Carlo Simulation and a mixed integer linear optimisation to plan extension of an existing grid. For simulation of grids with a high number of nodes Tomim [22] uses a separation of the total grid according to the Multi Thévenin Equivalents (MATE) and simulates it on distributed systems. Rudion [20] developed a model describing the total behaviour of a wind park. This reduced model has got the same electrical behaviour at the point of connection as the detailed modeled windpark. This paper picks up these ideas and derives equivalent models of low voltage grids.

An interesting approach is to aggregate grid segments where no restrictions must be regarded and to derive an equivalent model describing the behaviour at the point of connection. The easiest model is to aggregate all components into one node and to neglect all other components, like cables, within the subgrid. The rate of reduction will be very high, but no internal effects can be modelled. This disadvantage is avoided by the approach of symbolic model reduction. Within this paper the approach is used to derive aggregated segements of distribution grids.

A. Symbolic Model Reduction

The idea of symbolic analysis and model reduction originates from the area of analog circuit design [13]. Recently this approach has been successfully applied to other physical domains such as hydraulic systems or mechatronical systems [15]. Within the project NetMod (http://www.netmod.org) funded by the German Federal Ministry of Education and Research (BMBF), we transferred the modelling approach to energy distribution grids. The approach of symbolic model reduction starts with the detailed model of the energy system. The physical constraints are defined by equations of the different grid components. For the reduction process the symbolic representation of the model parameters and variables is essential. Fig. 2 shows the principal of the symbolic reduction approach. The detailed model is analyzed with numeric methods and variables that have only little influence to the desired output variables are removed. Aggregation of the remaining equations results in the reduced model. In mathematical terms the detailed nonlinear system of equations is being converted into a nonlinear system with lower problem dimension equivalent at the numeric input/output behaviour.

In the first step the set of equations can be reduced without any reduction error. Besides, no numeric error will occur. These steps are

- algebraic reductions,
- branch reductions.
In case of algebraic reductions only algebraic conversions, such as term canceling due to linear dependencies, are applied. For example the set of equations

\[
\begin{align*}
    x_1 &= y_1 + y_2 \\
    x_2 &= y_2 + y_3 \\
    x_3 &= x_1 - x_2 + y_3
\end{align*}
\]  

(2)

can be replaced by:

\[
x_3 = y_1
\]  

(3)

Branch reductions reduce the complexity of equations which are defined by sections. If a section is not used during the whole simulation - it can be neglected without any error. For example the function \( y \) defined by sections

\[
y = x_1 + \begin{cases} 
    x_1 \cdot x_2 & \text{, if } x_2 > 1 \\
    \frac{1}{2} x_1 \cdot (1 + x_2^2) & \text{, if } x_2 < 1
\end{cases}
\]

(4)

can be reduced to

\[
y = x_1 + x_1 \cdot x_2
\]

(5)

if \( x_2 > 1 \) is valid for the total simulation.

A further reduction of the equation set is possible if approximating reduction steps are used. These techniques will influence the numeric result of the simulation so that the error must be controlled. The error must be limited by definition of input and output variables as well as by setting a maximal error for the desired output variables. Three approximating reduction techniques are possible:

- switch reductions
- term replacing
- term canceling

Switch reductions can switch physical effects on or off. If a certain physical effect does not influence the desired input/output behaviour dramatically it can be neglected resulting in only small errors. In case of term replacing a non static term is replaced by its mean value. In case of term canceling terms that do not influence the desired variables are removed. After applying each possible approximation and non approximation reduction steps a symbolic model approximating the detailed model results. This model still contains original variables and parameters and can be used for analyzing of the system. The advantage of this automatic reduction approach is its simplicity meaning that only the model must be created and no expert knowledge for reducing the model complexity is necessary.

The introduced technique of symbolic reduction is available in the Add-On Analog Insydes [1] for the computer algebra system Mathematica [3]. This is a platform allowing the calculation of symbolic equations. Time critical functions are written as an external C/C++ code. Analog Insydes uses a special syntax to define netlists of electricity grids.

B. Showcase: Symbolic model reduction in a rural grid

The so far introduced method of symbolic reduction should be used to reduce segments of an electricity distribution grid, described by loadflow equations (1). The model of this sub

![Fig. 3. For evaluating the potential of model reduction three scenarios of different complexity (detailed, symbolic, and reduced to one node) have been analyzed. The different grids are connected to a medium voltage grid that provides the nominal voltage \( V_{nom} = 20 \, \text{kV} \) at the input of each grid. The power balance of the three scenarios will be evaluated.](image)

grid must be able to calculate the current \( I_{sub\,grid} \) at the point of connection of the sub grid

\[
I_{sub\,grid} = f \left( V_{in}, P_{load}, P_{gen} \right)
\]

(6)

depending on the input voltage \( V_{in} \). Within the sub grid generation \( P_{gen} \) and load \( P_{load} \) are set by external parameters. To integrate such sub grid models in the whole system two constraints must be fulfilled:

- connectivity constraint
- approximation constraint

The connectivity constraint implies that the interface of the reduced and the original model are equal. This is fulfilled by the desired input/output behaviour of the model through defining the current at the point of connection depending on the node voltage. Hereby, the approximation constraint postulates to keep the output behaviour within a certain error bound. This is achieved by setting a maximal error for the current at the point of connection.

To evaluate the approach of symbolic reduction for electricity distribution grids a typical low voltage grid is modeled using three steps of complexity (Fig. 3). Therefore, a low voltage feeder of a typical rural grid with distributed energy generation has been chosen as an example for the application analyzing the model reduction approach. The system contains 4 photovoltaic units (4 kW each), a biomass cogeneration plant (60 kW) as well as a wind turbine with a nominal power of 1.2 MW close to the transformer. The grid also supplies 4 houses with a typical agricultural load profile. Each house is equipped with a connection power of 20 kW. The corresponding detailed mathematical model, described by the loadflow equations (1), consists of 18 grid nodes and will be used as reference case (scenario: detailed).

After application of symbolic model reduction this system can be automatically reduced from originally 228 to 26 remaining equations where a model accuracy of 10 mA has been chosen for the grid currents at the point of connection. Moreover, the reduced system still includes the parameter dependencies (generation and loads) of the original system. This symbolical reduced model was integrated as an efficient C-code into the developed simulation framework, which also includes loadflow analysis. This scenario will be regarded on the second complexity level (scenario: symbolic).
The third complexity step is described by using a manually reduced model. The entire PV generation, loads, and CHP generation are connected at the end of the medium voltage line connecting the rural grid. The low voltage transformer will only have little influence and will therefore be neglected. The medium voltage line with a capacitor of $C' = \frac{235 \text{ kF}}{\text{kV}}$ will have great influence on the loadflow and will be kept in the model. Finally this so called one node reduction scenario (one node) is represented by a model containing 3 grid nodes. It will have the strongest reduction factor, but no internal grid effects such as losses are modeled. This is also a manual method always requiring expert knowledge about grid behaviour to derive it.

The three defined grids (Fig. 3) are connected to a medium voltage grid which provides the nominal voltage of $V_{in} = 20 \text{ kV}$ at the input. Within the simulation study the power of generators and loads have been varied to analyze stability and quality of the grid models. The scenarios are evaluated according to the power balance at the point of connection and its solving time.

Fig. 4 shows the relative error in the power at the point of connection for active $\Delta P$ and reactive $\Delta Q$ power of the rural grid for varied household loads is shown.

Finally, the solving time has been analyzed and visualized in Fig. 5. With both reduction methods a reduction of simulation time can be expected. With the aggregation of the grid to one node the grid reduces from 18 to 3 grid nodes which reduced simulation time from $78 \frac{\text{ms}}{\text{w}}$ to $46 \frac{\text{ms}}{\text{w}}$ for each time step $\Delta t$. This corresponds to a reduction factor of about 40 %. The symbolic model only consists of one grid node, but for each iteration a set of equations with 26 equations must be solved. This results in a simulation time of $51 \frac{\text{ms}}{\text{w}}$ corresponding to an acceleration factor of 34 %. Also the accuracy stays within a range smaller than 1 % compared to the detailed model. Further acceleration can be seen in larger grids with a higher number of nodes because solving time of loadflow analysis increases disproportionately with the number of nodes. This can be reduced by replacing grid segments using equivalent equations.

### III. REACTIVE POWER CONTROL IN DISTRIBUTION GRIDS

As mentioned above, with a rising share of fluctuating generation in the electricity grids new control strategies for grids must be developed. Besides, the day-ahead scheduling of controllable loads and generators using time variable tariffs reactive power control strategies are discussed. Whereas reactive power control was a domain of high voltage level, the new German guidelines for connection of generators to medium voltage levels [2] give grid operators the possibility to provide a certain behaviour for feeding reactive power into the grid. It can be differentiated into dynamic and static control. The dynamic control works in case of errors and forces decentralized generators to feed-in short circuit power for a duration of 150 ms after a failure to stabilize the grid. If the failure is still measured after this time span the generator must shut down. This behaviour is often called “fault-ride-through”. This paper will concentrate on static control keeping voltages and power flows within normal operation mode. Several control strategies for the power factor $\cos \varphi$ or reactive power $Q$ in dependence of the actual voltage $V$, the actual power $P$, or the relative deviation of the voltage $\Delta V = \frac{\Delta V}{V_{ref}}$ from the reference voltage $V_{ref}$ are defined:

- fix power factor: $\cos \varphi_{fix}$
- reactive power depending on of power: $Q(P)$
• reactive power depending on of voltage: \( Q(V) \)
• phase angle depending on voltage deviation: \( \varphi(\Delta v) \)

A. Showcase: reactive power control in distribution Grids

The following section discusses reactive power control on a characteristic distribution grid. Fig. 6 shows the medium voltage which is used for this case study. To achieve representative results it is necessary to use a grid scenario fulfilling the targets of future “Smart Grids” with a high amount of fluctuating generation. The reference grid defined by the International Council on Large Electric Systems (CIGRE) [7] is one benchmark grid fulfilling these recommendations. For the presented showcase the reduced CIGRE benchmark grid of the project NetMod will be used [14]. According to typical settlement structures different sub grids are connected to the nodes of the CIGRE grid. For each node Fig. 6 shows which sub grids are connected. Originally, the sub grids are defined in [11]. TABLE I shows the main components of the different sub grids.

![Diagram of medium voltage lines of CIGRE-Benchmark grid](image)

**Fig. 6.** The medium voltage lines of the CIGRE-Benchmark grid are shown. At each node the number and type of connected sub grids is given.

By implementing the loadflow analysis of the CIGRE grid including the detailed sub grid models takes about 25 \( \Delta t \) per timestep \( \Delta t \). The simulation of a day with 96 timesteps will take about 40 minutes. For implementing a reactive power controller for each timestep several internal iterations are necessary. Using the detailed model this would result in very long calculation times. So subgrid models with only small decentralized generators (sub grids: town and rural) were transformed by using the method of symbolic model reduction, where a maximal error of 10 mA for the current at the connection point was set. The resulting model contains detailed models for industry and business sub grids. Rural and town sub grids were taken as reduced models. Solving this partly reduced model results on the same computer with a solution time of 6 \( \Delta t \) corresponding to an acceleration of almost one order of magnitude. But the result will stay in the guaranteed error band.

This symbolically reduced model has been used to implement a reactive power controller for the cogeneration plants in the industrial sub grid. If voltage at the connection point exceeds nominal voltage \( V_n \) it should be reduced by absorbing inductive reactive power. This implies that the phase angle \( \varphi \) of the complex power must be greater than zero. This is done by an internal iteration loop for the controlled generators. While the relative deviation of the node voltage \( \Delta v = \frac{\Delta V}{V_0} \) is greater than 0.001 \( V_n \) the phase angle of the power \( \varphi_{n-1} \) for loop \( n \) is calculated by:

\[
\text{while } (|\Delta v| > 0.001V_n) \land (n \leq \text{maxIter})
\]

\[
\varphi_n = \varphi_{n-1} + \Delta \varphi(\Delta v)
\]

(7)

If also the actual iteration step \( n \) does not exceed the maximal number of internal iterations \( \text{maxIter} \) the phase angle \( \varphi_{n-1} \) is calculated as the sum of the angle at the last iteration step \( \varphi_{n-1} \) and \( \Delta \varphi(\Delta v) \). The changing of the phase angle

\[
\Delta \varphi(\Delta v) = \frac{|\Delta U|}{|Z_{line}| \cdot |L_G|} = \Delta v \frac{\pi}{2000}
\]

(8)

depends on the line impedance \( Z_{line} \) and the feed-in current \( L_G \). With the values of the grid and a typical feed-in current the factor \( \frac{|\Delta U|}{|Z_{line}| \cdot |L_G|} = 0.005 \) is used for the controller. If the power factor \( \cos \varphi \) is smaller than 0.85 voltage cannot be controlled anymore. In this case the phase angle \( \varphi \) is kept constant.

The control algorithm defined in (7) has been implemented for the cogeneration plants (CHP) in the industry sub grids. In total these were 10 generation units with a total nominal power of 3 MW. After simulation of the control algorithm the voltages on the medium voltage lines of the CIGRE grid have been analyzed for a time period of one week in winter. The load and generation profiles were chosen as typical profiles in Freiburg im Breisgau in the very south of Germany. Fig. 7 shows the load and generation balance including the cogeneration schedules for the simulated week. It can be seen how the load curve is covered by the sum of fluctuating generation (wind and PV) and cogeneration. The difference is feed-into the grid or is imported.

**TABLE I**

<table>
<thead>
<tr>
<th>sub grid</th>
<th>components</th>
</tr>
</thead>
<tbody>
<tr>
<td>town</td>
<td>typical grid with household loads connecting 8 feeders connecting 112 households, 28 PV-plants (4 kW, each), 28 micro CHP (2 kW, each)</td>
</tr>
<tr>
<td>rural</td>
<td>typical grid with rural loads connecting 1 feeder connecting 4 households, 4 PV-plants (1 kW, each), 1 CHP (60 kW)</td>
</tr>
<tr>
<td>wind farm</td>
<td>5 wind turbines with a nominal power of 720 kW</td>
</tr>
<tr>
<td>business</td>
<td>typical grid with small business consumers connecting 4 feeders connecting 36 consumers, 1 CHP with a nominal power of 200 kW</td>
</tr>
<tr>
<td>industry</td>
<td>typical grid with industry consumers connecting 4 feeders connecting 4 consumers, 2 CHP with a nominal power of 300 kW</td>
</tr>
</tbody>
</table>

The main components of the low voltage sub grids that are connected to the CIGRE benchmark grid are defined according to [11].
The distribution of the voltages on the 11 medium voltage nodes of the CIGRE grid are shown in the bar plot in Fig. 8. At most times generation exceeds load (Fig. 7) which results in mean voltage slightly above the nominal voltage of $V_n = 20$ kV. Without reactive power controllers the voltage for all nodes is within an interval of $V \in [19.9 \text{ kV}, 20.8 \text{ kV}]$ with a mean value of about 20.4 kV. With a reactive power controller the voltage span reduces to $V \in [19.9 \text{ kV}, 20.6 \text{ kV}]$ with a mean value of about 20.3 kV. If the reactive power control is applied the span of voltage variations and also the absolute value decreases.

The results of the simulation show that it is possible to reduce the band of voltages and the absolute value of voltages in distribution grids. The span of voltages offers more flexibility in the operation of the so called “Smart Grid”. For example, by using tap changers the voltage can be shifted easily. This and the fact that the voltage level decreases offers the possibility to connect further distributed generation within the grid.

IV. CONCLUSION

The simulation studies presented within this paper primarily showed the gain of the symbolic reduction approach in electricity distribution grids. By reducing the complexity of sub grids where no grid restrictions must be regarded the simulation time of a typical distribution grid could be reduced by nearly one magnitude. These complexity reduced models offer the possibility to focus on controllable devices e.g. to control reactive power. By controlling 10 cogeneration units with a total power of 3 MW within the simulation scenario the mean value of the voltage of the medium voltage grid could be reduced from 20.4 kV to 20.3 kV. Model based grid calculation offers the optimisation of operation of “Smart Grids”. Voltage leves as well as grid efficiency (reactive power) will be managed by this method. By including of also solar inverters and a higher share of renewable generators the effect will increase.

REFERENCES


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Bernhard Wille-Haussmann

Bernhard Wille-Haussmann, born in 1977, studied electronic engineering at the University of Stuttgart. In his diploma thesis (finished 2005) he worked on the characterisation of thin film photovoltaics. Since 2005 he works on his PHD at Fraunhofer ISE in the field of managing distribution grids with a high penetration of renewable and distributed generators. His main task is the optimal operation of heat systems with cogeneration and thermal storage devices.

Bernhard Wille-Haussmann’s projects focused on simulation of energy systems. Within the project NetMod mathematical reduction techniques have been analysed and developed to model electricity distribution grids more efficient. These results have been applied in the project Virtplant which in cooperation with the local utility builded up a regional virtual power plant in the city of Freiburg.

Jochen Link

Jochen Link, M.Sc: born 1980, studied “Renewable Energy Systems” for his Bachelor and Master’s Degree at FHTW Berlin. Since 2006 he has been working at Fraunhofer ISE in the fields of control devices for distributed energy systems. He is currently working on his PhD with the topics grid integration of electric mobility and decentralised energy systems. His PhD research is supported by Reiner-Lemonie Foundation

Christof Wittwer

Dr. Christof Wittwer was born in 1967 and studied electrical engineering at the University of Kaiserslautern. He has been working at the Fraunhofer Institute for Solar Energy Systems ISE since 1992 and concentrated his activities on thermal simulation and control in various R&D projects in the beginning of his research activities. Between 1996 and 1999 he worked as a research assistant in the fields of building services engineering and systems control receiving his promotion in 1999 at the University of Karlsruhe. His research activities focused on the development of a dynamic system simulator which is used for the control deployment of thermal energy systems. Since 1999 Christof Wittwer has been working as a project manager at Fraunhofer ISE concentrating on research and development of networked control systems for decentralised power generation systems. Since 2003 he is leading the group “Control Devices BSR” at Fraunhofer ISE which focuses on decentralised energy systems and grid integration.

Christof Wittwers national and international systemtechnology projects aim on smart grid and smart metering topics as they integrate software and hardware for controlling systems, energy management and operation control systems to integrate local producers, storages, and loads, virtual power plants, intelligent load control (Demand Side Management), grid services, and smart metering systems. Selected projects are Dispower which focused on development of technical concepts and tools, eTelligence in the E-Energy model region Cuxhaven where a smart grid currently is developed including a market platform for energy services as well as DEMAX which aims on the development and test of flexible electricity tariffs and local energy management. Another current project is MASSIG where marketing concepts for small local producers are developed.