PMU-Based Multi-Area State Estimation with Low Data Exchange

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Abstract—The paper proposes a novel multi-area state estimator based on wide area measurements where only boundary buses are considered in the coordination level. The power injection measurements are not used during the coordination level reducing the number of states and therefore size of the problem. Instead, a set of pseudo-measurements are included whenever a power injection measurement is available in boundary buses. The proposed method was validated using the IEEE 300 bus system split into 7 different areas.

Index Terms—Distributed State Estimation, Measurement Uncertainty, Multi-Area State Estimation, Pseudo-Measurements, Synchronized Measurements.

I. INTRODUCTION

INTEGRATED state estimators are the most accurate option when estimating the state of a power system. They make use of all available measurements in the entire power network to estimate the system operating point. However, the size and complexity of large power networks suggests that a strategy of decentralizing the estimation problem by distributing the computation across the system, rather than centralizing it in a single control centre, may be beneficial [1]. This strategy is the main idea of Distributed State Estimators (DSE) or Multi Area State Estimators (MASE). They provide reliable estimates for large scale power systems with significantly reduced computational requirements when compared to the aforementioned integrated solutions.

A good MASE must fulfill the following basic requirements: a) minimal computer requirements, b) high accuracy, c) robustness, d) bad data processing and e) minimal data transfer [2].

It is important to obtain accurate estimates of the actual power transfers between areas as power transaction operations will rely on the information given by the state estimator.

Some efforts have been made to obtain the same, or very similar, accuracy as the integrated solution. Reference [3] introduces an optimization method that reaches a wide-area sub-optimal solution by solving local area optimization problems. The Diakoptic based distributed estimator proposed in [4] is able to obtain the same accuracy as the integrated solution. In addition, the method proposed in [5] includes a set of virtual measurements to obtain consistent solutions in boundaries of different areas. However, the disadvantage of these methods is the data dependency among areas during the estimation process.

At present, two-level estimators are the most common approach for MASE due to their simplicity and limited data exchange between areas. In two-level MASE, the power system is separated into small observable subsystems, each one assigned with a slack (reference) bus to run a lower (local) level state estimation.

The state estimation solution of each subsystem is then collected by a central coordinator which unifies and coordinates the lower level solutions in order to obtain an overall estimate of the entire power system. This stage is known as the higher or coordination level.

The availability of Phasor Measurement Units (PMU) improves the accuracy and redundancy of existing state estimators as direct measurements of phasor angles are extracted from the synchronization of PMUs across the system [6], [7]. In the case of MASE, these synchronized measurements can also be used to improve bad data processing around boundary buses [8] or to measure the phase shift between the slack buses of different areas [9].

One challenge in two-level estimators, particularly during the coordination level, is how to deal with power injection measurements in boundary buses. If they are included, each area will have to provide some information about their internal topology to the coordination level [8]. However, this option might not be always feasible as utilities usually prefer to restrict their topology configuration information for privacy and/or security reasons. In addition, considering power injection measurements makes it necessary to include the states corresponding to internal buses adjacent to boundary buses, increasing the size of the coordination level.

Another option would be not including the power injection measurements of boundary buses; this would however result in a loss of information and lower redundancy at the coordination level.

The methodology presented in this paper avoids the use of power injection measurements in boundary buses. This reduces the data exchange between local and coordination estimators. Instead, a new set of transferred powers pseudo-measurements are included to maintain the redundancy level and accuracy of the coordination level. Moreover, wide area measurements obtained from PMUs are used in boundary buses and slack buses, improving the efficiency of both the lower (local) and the higher (coordination) estimation levels. The proposed PMU based estimator eliminates the interaction of local area estimators and minimizes the data exchange between local and coordination estimators. This practical,
robust and effective estimator was validated through computer simulated tests using standard IEEE test systems.

II. LOCAL AREA STATE ESTIMATORS

Each local area state estimator provides an estimate of the sub-system state based on the available measurements in each area. For any area \( i \), the following three bus types can be identified:

- Internal bus: any bus that is not adjacent (connected) to any external bus.
- Boundary bus: any bus adjacent to at least one external bus. The interconnection between a boundary bus and an external bus is referred to as a tie-line.
- External bus: any bus belonging to a different area that is connected to at least one boundary bus of area \( i \), by a tie line.

Let us define a set of measurements \( z_i \) in area \( i \), as follows:

\[
z_i = h(x_i) + e_i \tag{1}
\]

where \( h(x_i) \) is the set of non-linear equations relating the measurements with the state variables \( x_i \), \( e_i \) is a set of uncorrelated measurement errors with Gaussian distribution.

The state vector is defined as follows:

\[
x_i = [x_{i}^{\text{int}}, x_{i}^{\text{b}}, x_{i}^{\text{ext}}]^T \tag{2}
\]

where:

- \( x_{i}^{\text{int}} \) is the set of bus voltages corresponding to the internal buses of area \( i \).
- \( x_{i}^{\text{b}} \) is the set of bus voltages corresponding to the boundary buses of area \( i \).
- \( x_{i}^{\text{ext}} \) is the set of bus voltages corresponding to the external buses of area \( i \).

The best estimation of the system states in area \( i \) is obtained through the Weighted Least Square (WLS) method, when minimizing the function \([10]\):

\[
J_i(x) = \left[ z_i - h(x_i) \right]^T \cdot \mathbf{R}^{-1} \cdot \left[ z_i - h(x_i) \right] \tag{3}
\]

where \( \mathbf{R} = \text{diag}\{\sigma_1^2, \sigma_2^2, ..., \sigma_m^2\} \) and is the error-covariance matrix containing information of measurement weights, expressed in terms of the square of the inverse of standard deviation \( \sigma \) for all \( m \) measurements. At iteration \( k \), the best estimate is:

\[
\Delta x_i^k = \mathbf{G}(x_i^k)^{-1} \cdot \mathbf{H}(x_i^k) \cdot \mathbf{R}_k^{-1} \cdot \left[ z_i - h(x_i^k) \right] \tag{4}
\]

where \( \mathbf{G}(x_i^k) = \mathbf{H}(x_i^k)^T \cdot \mathbf{R}_k^{-1} \cdot \mathbf{H}(x_i^k) \) is the Gain Matrix and \( \mathbf{H} \) is the Jacobian matrix. The iteration procedure stops when \( \Delta x_i^k = x_i^{k+1} - x_i^k \) is smaller than a pre-defined value. For a power system with \( S \) areas, the above procedure must be carried out in parallel for all \( i = 1, 2, ..., S \).

A more robust state estimator can be achieved by including the equality constraints for null power injection buses. The minimization problem stated in (3) is now extended to meet a set of constraints \( e(x_i) = 0 \). The solution is obtained by applying the iterative procedure:

\[
\begin{bmatrix}
\Delta x_i^k \\
\lambda_{k+1}
\end{bmatrix} = 
\begin{bmatrix}
\mathbf{H}^T \mathbf{R}_k^{-1} \mathbf{H} & -\mathbf{C}^T \\
-\mathbf{C} & 0
\end{bmatrix}^{-1} 
\begin{bmatrix}
\mathbf{H}^T \mathbf{R}_k^{-1} (z_i - h(x_i^k)) \\
e(x_i^k)
\end{bmatrix} \tag{5}
\]

where \( \lambda \) is the vector of Lagrange multipliers and \( \mathbf{C} = \partial e(x_i)/\partial x_i \).

The use of constraints in the estimation procedure is similar to including measurements with large confidence. Each local estimator must have enough measurements to make the system fully observable with redundant measurements to detect and eliminate bad data. Each area has its own reference; hence, there are \( S \) different slack buses in the interconnected power network, one for each area.

Based on the principle that each local estimator is independent of any other estimator (and vice versa), all synchrophasor measurements in area \( i \) will be referred to its local reference in the lower level estimation.

Fig. 1 shows how the PMU measurements are referred to its own slack bus during the lower level estimation. Without loss of generality, it can be assumed that a PMU is located at the local slack bus. In fact, installing a PMU in each slack bus will improve the coordination level estimation, as will be explained later.

III. COORDINATION LEVEL

The higher (coordination) level estimator uses the estimated states corresponding to boundary buses, obtained from local estimators, and those measurements at the boundaries of each area to create the set of measurements \( z_c \):

\[
z_c = h(x_c) + e_c \tag{6}
\]

The WLS method is used to estimate the new set of states \( x_c \) at the coordination level. This vector is defined as:

\[
x_c = [x_i^b, \theta_i^k]^T \ \forall \ i = 1, 2, ..., S \tag{7}
\]

where:

- \( x_i^b \) is the set of boundary bus voltages in area \( i \). In the coordination level all bus voltage angles are referred to the global slack bus.
- \( \theta_i^k \) is the slack bus angle for area \( i \) referred to the global slack bus.

The set of measurements \( z_c \) stated in (6) is defined in detail in the following Sub-Sections:

A. Synchronized Measurements

By including PMU measurements in MASE, the accuracy of the overall estimation (lower level and coordination level) is improved. Firstly, a PMU improves measurement redundancy levels and the estimation accuracy of individual subsystems. Secondly, if PMUs are located at the boundaries of the subsystems, they will also improve the accuracy of the coordination level \([8]\). In addition, when PMUs are located at the slack bus of each subsystem, the angle difference estimation between slack buses will be determined more easily at the coordination level.
The synchronized measurements used in the Coordination Level can be separated in the following measurement vector:

\[ z_{\text{sync}}^{i} = \left[ \theta_{i}^b, V_{i}^b, I_{i}^r, I_{i}^i, \theta_{i}^k \right]^T, \quad \forall i = 1, 2, ..., S. \tag{8} \]

where:
- \( \theta_{i}^b \) is the set of boundary bus voltage angle measurements in area \( i \) referred to the local reference in area \( i \).
- \( V_{i}^b \) is the set of boundary bus voltage magnitudes in area \( i \).
- \( I_{i}^r \) and \( I_{i}^i \) are the real and imaginary part of measured currents from area \( i \) to area \( j \). These phasor measurements are also referred to the local reference in area \( i \).
- \( \theta_{i}^k \) is the slack bus’s angle measurement in area \( i \) referred to the global reference.

The PMU measurements in boundary buses remain with the same reference (local slack bus), whilst angle measurements of slack buses will be referred to the global reference (global slack bus), as presented in Fig. 1.

Based on this, the voltage angle measurement \( \theta_{i}^k \) in the boundary bus \( k \) of area \( i \) is represented in \( h(x_c) \) as follows:

\[ \theta_{i}^k \rightarrow h(x_c) = \theta^k - \theta_{i}^k \tag{9} \]

The real and imaginary components of the current measurement from bus \( i \) (in area \( i \)) to bus \( j \) (in area \( f \)) is represented as:

\[ I_{i}^{ab} = (g_{i} + g_{a})V_{i}^{c} \cos(\theta - \theta_{i}^{a}) - (b_{i} + b_{a})V_{i}^{c} \sin(\theta - \theta_{i}^{a}) \]
\[ + V_{j}^{c} (b_{i} \sin(\theta - \theta_{i}^{a}) - g_{i} \cos(\theta - \theta_{i}^{a})) \]
\[ I_{i}^{ad} = (g_{i} + g_{a})V_{i}^{c} \sin(\theta - \theta_{i}^{a}) + (b_{i} + b_{a})V_{i}^{c} \cos(\theta - \theta_{i}^{a}) \]
\[ - V_{j}^{c} (g_{i} \sin(\theta - \theta_{i}^{a}) + b_{i} \cos(\theta - \theta_{i}^{a})) \tag{10} \]

where \((g_{i} + j/b_{i})\) is the shunt admittance connected at bus \( i \), and \((g_{ij} + j/b_{ij})\) is the series admittance of the tie-line connecting area \( i \) and \( j \).

It is important to note that the slack bus angle of the area where the PMU is located must be included in the model because the PMU measurements are still referred to their local reference. Finally, the slack bus angle of area \( i \) is already referred to the global reference and therefore does not need any correction of reference.

### B. Conventional Measurements

All transferred power measurements in tie lines will be used by the coordination level. However, power injection measurements in boundary buses will not be used as it would be necessary to share information about internal topology of the areas. In addition, the size and complexity of the coordination level would increase.

Each area will provide minimum information about its internal topology configuration for security reasons. Thus, whenever an injected power measurement is found (at a boundary bus), it will be replaced by the estimated transferred power, in the relevant tie line, obtained by the local estimator.

The injected power measurements of boundary buses were used in the lower level estimation, therefore it is reasonable to believe that the estimated states related to these measurements are sufficiently accurate. Thus, the estimated tie-line flows can be used as an effective way to maintain redundancy in the coordination level.

### C. Pseudo-Measurements

The estimated bus voltages of all boundary buses \( \hat{x}_{b} \) will be used as pseudo-measurements in \( z_{c}. \) The inverse of the Gain Matrix (covariance matrix of the estimated states) of the

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**Fig. 1. Multi-Area power system with PMU measurements for state estimation (local level and coordination level)**
local estimators will be used to weight the pseudo-
measurements at the coordination level:

\[ P_s = \text{diag}(H^T R_s^{-1} H)^{-1} \]  \hspace{1cm} (12)

In addition, the covariance matrix of the new pseudo-
measurements (corresponding to estimated transferred powers)
can be approximated as:

\[ P_{sm} = H_{sm} P_s H_{sm}^T \]  \hspace{1cm} (13)

where \( H_{sm} \) contains the partial derivatives of the transferred
powers in tie lines with respect to the states corresponding to
boundary buses. Thus, the vector of measurements \( z \) in the
coordination level consists of:

\[ z_c = [z^b_i, z^p_i, \hat{x}^b_i]^T \quad \forall \ i = 1, 2, ..., S \]  \hspace{1cm} (14)

where:
- \( z^b_i \) is the set of conventional and synchronized measurements
  in the boundary buses for area \( i \)
- \( z^p_i \) is the set of estimated transferred powers whenever a
  boundary power injection measurement is found in area \( i \)
- \( \hat{x}^b_i \) is the set of estimated bus voltages in the boundary buses
  of area \( i \).

Fig. 2 shows the lower and higher level schemes for a
power system with \( S \) areas. Each independent local estimator
calculates the set of bus voltages \( x_i \) (internal, boundary and
external buses connecting area \( i \)).

The coordination level does not use any information
regarding the internal topology of the areas.

The estimation for the coordination level is obtained
through an iterative procedure:

\[ \Delta x^c = G(x^c)^{-1} \cdot H^T (x^c) \cdot R_c^{-1} [z_c - h(x^c)] \]  \hspace{1cm} (15)

where \( R_c = \text{diag} \{ \sigma_1^2, \sigma_2^2, ..., \sigma_m^2 \} \) is the error-covariance matrix
of \( z_c \).

IV. SIMULATIONS

The proposed multi area state estimator with minimum data
exchange has been tested by using the IEEE 300 bus system
and it was compared with other estimators for validation
purposes.

The 300 bus system was arbitrarily split into 7 (\( S = 7 \))
different areas, as described in Table I.

<table>
<thead>
<tr>
<th>Area</th>
<th>Buses</th>
<th>Branches</th>
<th>Boundaries</th>
<th>To Areas</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>101</td>
<td>129</td>
<td>16</td>
<td>2, 3, 4, 5, 7</td>
</tr>
<tr>
<td>2</td>
<td>45</td>
<td>56</td>
<td>10</td>
<td>1, 5</td>
</tr>
<tr>
<td>3</td>
<td>42</td>
<td>57</td>
<td>5</td>
<td>1, 4</td>
</tr>
<tr>
<td>4</td>
<td>45</td>
<td>62</td>
<td>4</td>
<td>1, 3</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
<td>53</td>
<td>8</td>
<td>1, 2, 6</td>
</tr>
<tr>
<td>6</td>
<td>35</td>
<td>41</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>36</td>
<td>38</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

A power flow solution of the 300 bus system was used to
obtain the set of measurements. The measurements were
corrupted with additive random Gaussian noise, used to
simulate the natural noise existing in real measurements.

A. Lower Level

The constrained WLS methodology was used to estimate
the states of each area and the equality constraints were
included to deal with any null power injections. The set of
noisy measurements consists of conventional and
synchronized measurements with the corresponding standard
deviation shown in Table II.

<table>
<thead>
<tr>
<th>Conventional</th>
<th>Synchronized</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage Mag.</td>
<td>Injected Power</td>
</tr>
<tr>
<td>0.2%</td>
<td>2%</td>
</tr>
<tr>
<td>Voltage Mag.</td>
<td>Power Flows</td>
</tr>
<tr>
<td>0.02%</td>
<td>2%</td>
</tr>
<tr>
<td>Voltage Mag.</td>
<td>Current Mag.</td>
</tr>
<tr>
<td>0.03%</td>
<td>Phase Angle</td>
</tr>
<tr>
<td>0.01°</td>
<td></td>
</tr>
</tbody>
</table>

The last column of the table indicates the threshold of the
Chi-Square Distribution test for \( m-n \) degrees of freedom and
confidence level of 95%. Here, \( m \) is the number of measurements and \( n \) the number of states.

### TABLE III
**Final Local Area Objective Function and Threshold for Chi-Square Distribution Test**

<table>
<thead>
<tr>
<th>Area</th>
<th>( m-n )</th>
<th>( J(\hat{x}) )</th>
<th>( \chi^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>473</td>
<td>472.0401</td>
<td>524.7022</td>
</tr>
<tr>
<td>2</td>
<td>226</td>
<td>197.4975</td>
<td>262.0697</td>
</tr>
<tr>
<td>3</td>
<td>258</td>
<td>243.4471</td>
<td>296.4659</td>
</tr>
<tr>
<td>4</td>
<td>289</td>
<td>279.6622</td>
<td>329.6489</td>
</tr>
<tr>
<td>5</td>
<td>210</td>
<td>196.2383</td>
<td>244.8076</td>
</tr>
<tr>
<td>6</td>
<td>162</td>
<td>138.6917</td>
<td>192.7001</td>
</tr>
<tr>
<td>7</td>
<td>177</td>
<td>151.1606</td>
<td>209.0424</td>
</tr>
</tbody>
</table>

Each estimator is free of gross bad data. Otherwise, it would be necessary to identify and eliminate the gross error in the set of measurements.

### B. Higher (Coordination) Level

The solutions of local area estimators and the measurements in boundary buses are considered in the coordination level. The following methods have been tested for comparison purposes:

**Method 1:** The coordination level includes internal buses adjacent to boundary buses so that the power injection measurements in the boundary buses can be used. The set of states are the bus voltage in the slack buses, boundary buses and internal buses adjacent to them. Therefore, it is necessary to know the internal connections of boundary buses. In addition, equality constraints have been included for those boundary buses with null power injections.

**Method 2:** This is the coordination level proposed in this paper. The set of states are bus voltages in slack buses and boundary buses only. Power injections measurements in boundary buses are not used in the coordination level. The estimated transferred powers flowing in or out the boundary bus are used as pseudo-measurements instead.

**Method 3:** Similar to Method 1 but constraints are not considered for null power injection buses.

Table IV gives a good overview of the size of the coordination level estimation according to the methods cited above. The set of measurements includes conventional and synchronized measurements. It is clear that including power injection measurements will significantly increase the size and complexity of the coordination level. The same applies for real interconnected power networks.

### TABLE IV
**Size of the Problem at Coordination Level**

<table>
<thead>
<tr>
<th></th>
<th>Method 1</th>
<th>Method 2</th>
<th>Method 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buses</td>
<td>115</td>
<td>48</td>
<td>115</td>
</tr>
<tr>
<td>Branches</td>
<td>113</td>
<td>25</td>
<td>113</td>
</tr>
<tr>
<td>( m-n )</td>
<td>225</td>
<td>235</td>
<td>197</td>
</tr>
</tbody>
</table>

The problem now is to check the accuracy of the simplified coordination level proposed in Method 2.

The overall estimation performance is presented in Table V based on the performance index calculated by:

\[
\sigma^2 = \sum_{i=1}^{N} (\hat{x}_i - x_i)^2
\]

where \( x_i \) is the true state obtained from the power flow calculation.

Since the number of states in Methods 1 and 3 is larger than Method 2 the states considered in (16) are those corresponding to boundary buses only.

### TABLE V
**Assessment of Coordination Level**

<table>
<thead>
<tr>
<th></th>
<th>Method 1</th>
<th>Method 2</th>
<th>Method 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>With PMUs</td>
<td>0.1127</td>
<td>0.1754</td>
<td>0.1567</td>
</tr>
<tr>
<td>No PMUs</td>
<td>2.6389</td>
<td>3.9942</td>
<td>3.5730</td>
</tr>
</tbody>
</table>

The results from Table V confirm that including PMUs in only a few boundary buses and all the slack buses improves the accuracy of the coordination level. Moreover, the performance index shows that excluding the power injection measurements in \( z_c \) only has a small impact on the accuracy of the coordination level.

The same effect was found when PMU measurements are not considered. Therefore, the price paid for excluding internal buses adjacent to boundary buses is relatively low when considering the benefits of simplicity, higher speed and reduced problem size of the coordination level.

The following results present a detailed study of the estimation errors for each bus in the coordination level. Fig. 3 and 4 show the absolute estimation errors for all boundary buses, excluding the null power injection buses.

The figures show that the inclusion of the pseudo-measurements of transferred powers gives a good approximation of power injection measurements but reducing the number of states. It is important to remember that Method 2 does not require information about internal connection of boundary buses and the final estimation is still similar to that of Methods 1 and 3.
The estimation errors for boundary buses with null power injection have been plotted in Fig. 5 and 6. It is clear that better estimates are obtained when equality constraints are included, see Method 1.

The estimation errors found when using Methods 2 and 3 are very similar to one another. Again, the advantage of Method 2 is the reduced data exchange from local area estimators to the coordination estimator.

C. Error in power injection measurement

The power injection measurement at bus 3 (boundary bus in area 1) was corrupted with wrong data in order to study its effect on the final estimation of the coordination level for the different methods. This error was undetected in the local area estimator and is therefore still present in the coordination level. Fig. 7 presents the absolute error of voltage magnitude and angle for Bus 3.

The estimation error for Bus 3 is lower when using the pseudo measurement of transferred powers which is calculated from the estimated states in the local estimator for Area 1. On
The other hand, the wrong measurement of injected powers affected the final estimation of Bus 3 and its surroundings as presented for Methods 1 and 3.

V. CONCLUSION

The paper presented a new Multi-Area State Estimator in which only boundary buses are considered in the estimation at the coordination level. The proposed methodology can be used when information regarding the surrounding boundary buses is unavailable at the coordination level.

The power injection measurements were replaced by pseudo measurements of transferred power to maintain redundancy in the coordination level and preserving valuable information from local area estimators.

The proposed methodology was found to deliver similar quality of results compared to those obtained when including internal buses adjacent to boundary buses, but with reduced size, computation times and complexity at the coordination level.

VI. REFERENCES


VII. BIOGRAPHIES

**Gustavo Valverde** (S’08) obtained the B.Sc degree in Electrical Engineering from the University of Costa Rica in 2005 and the M.Sc degree in Electrical Power Systems from the University of Manchester in 2008. Currently he is a PhD student at the University of Manchester working on state estimators using synchronized measurement technology. His research interests include monitoring of power systems, probabilistic analysis in power systems, state estimation, dynamics and voltage stability assessment.

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