SMALL-SIGNAL STABILITY ANALYSIS AND VSI CONTROLLER DESIGN FOR MICROGRIDS OPERATING IN GRID-CONNECTED MODE

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Abstract— A novel approach for VSI with an LCL output filter controller design for microgrids operating in grid-connected mode is presented in this paper. The controller structure is based on a multiloop control configuration in which the inner control loop is responsible for current regulation and the outer control loop generates the reference currents based on preset active and reactive power injection levels. In order to evaluate the microgrid small-signal stability, a linearized model of the microgrid is used. The modeling approach adopted in this paper is intrinsically modular, in the sense that models for the system components are individually derived and then aggregated into a single microgrid model. The small-signal model includes the representation of fast dynamic phenomena that arise with the widespread use of electronically interfaced microsources typical of microgrid applications. Therefore, the dynamics of (i) Voltage Source Inverters (VSIs) and companion LCL output filters; (ii) VSI controllers; (iii) loads, and (iv) the electrical network, can all be taken into account. The small-signal stability analysis results are then validated through time-domain nonlinear simulations performed with the PSCAD/EMTDC software package.

Keywords— Microgrids, Small-signal stability, Voltage source inverter (VSI), Network dynamics.

Palavras-chave— Microredes, Estabilidade a pequenos sinais, Inversor fonte de tensão, Dinâmica da rede elétrica.

1 Introduction

Environmental, economical, technological and social impacts related to large scale power generation and transmission has led to the use of distributed energy resources (DER) in the electric power systems. However, limitations related to the widespread use of distributed generation (DG) in passive distribution networks has led to the more complex concept of microgrids (Lasseter, 2002).

A microgrid comprises a group of radial feeders connected to the main grid through a point of common coupling (PCC). Microgrids can operate in two modes: grid-connected or islanded. In grid-connected mode, the microgrid imports or exports power according to its internal load and generation balance, and to proper market policies. In islanded mode, the microgrid is isolated from the main grid and the load is supplied only by local microsources (Lasseter, 2002).

Microsources are generally connected to the microgrid through power electronic interfaces (Lasseter, 2002). The most used interface device is the three-phase pulse-width modulation (PWM) voltage-source inverter (VSI).

An important characteristic of microsources is their low or even null inertia (Chowdhury et al., 2009), which makes the dominant microgrid dynamics lie in the range of frequencies higher than the classic electromechanical oscillations of conventional power systems. Therefore, it is necessary to derive mathematical models that take into account the fast dynamic behavior of microgrid components, such as VSI output filters and the network.

To perform microgrid small-signal stability analysis and VSI controller design, it is necessary to derive a linearized microgrid model considering the VSIs dynamics, their respective controllers, and the network. This issue has already been investigated by Katiraei et al. (2007), where the VSI output filter is a simple L filter. Rese et al. (2012b) performed this task considering more effective LCL filters, which also exhibit more complex dynamics.

The objective of this paper is to propose a novel control structure and a design procedure for VSIs with an LCL output filter, applicable to microgrids operating in grid-connected mode. Fur-
thermore, a brief description of a microgrid small-signal model for grid-connected operating mode, originally proposed by Rese et al. (2012b), is presented. This linearized model is only used to perform VSI controller design and small-signal stability analysis of a microgrid test system. Results of small-signal stability analysis and controller design based on the developed linearized model are validated through nonlinear time-domain simulations performed with the PSCAD/EMTDC software package.

This paper is organized as follows. Section 2 presents the derivation of the microgrid small-signal model. Section 3 describes the proposed VSI controller structure for grid-connected operating mode. The methodology employed for VSI controller design is presented in Section 4. The small-signal stability analysis and VSI controller design for a specific microgrid test system, as well as time-domain nonlinear simulations results, are presented in Section 5. Section 6 presents some concluding remarks.

2 Microgrid Small-Signal Model

In this paper, the microgrid small-signal model is derived in a modular way so that the linearized state-space equations of each microgrid component can be developed individually, as done by Rese et al. (2012b). It is assumed that the microgrid operates in balanced conditions and that every microgrid component is represented in a $dq$ reference frame.

The description of VSI and network models are given in Subsections 2.1 and 2.3, respectively. The detailed equations of these models are given in (Rese et al., 2012a) and are not reproduced in this paper due to space limitations. The complete description of the VSI controller employed in this paper is given in Section 3.

2.1 VSI model

The general representation of a VSI connected to the network through an LCL filter is shown in Fig. 1. In this paper, the dynamic behavior of the DC side is not taken into account so that it can be represented by an ideal DC voltage source.

![Figure 1: Topology of a three-phase VSI with an LCL filter](image)

The LCL output filter is used to attenuate inverter switching harmonics. This filter is more adequate than an L and LC filters in microgrid applications because, in addition to providing higher high-frequency attenuation with the same inductance value, it is also suitable to operation in both grid-connected and islanded modes (Yang et al., 2011).

The VSI model is obtained via the State-Space Averaging Technique (Erickson and Maksimović, 2001). The resulting linearized model of the $k$-th three-phase VSI with an LCL output filter in the $dq$-frame is given by:

$$
\Delta \dot{x}_k = A_k \Delta x_k + B_{v,k} \Delta v_{dq,k} + B_{u,k} \Delta u_{dq,k}
\Delta y_k = C_{i,k} \Delta x_k + D_{i,k} \Delta v_{dq,k} + D_{c,k} \Delta u_{dq,k}
$$

(1)

where $x_k$ is the VSI state vector, $v_{dq,k}$ is the input grid voltage vector, $u_{dq,k}$ is the VSI control input vector, $y_k$ is the output vector (grid-side current), $A_k$, $B_{v,k}$, $B_{u,k}$, $C_{i,k}$, $D_{i,k}$ and $D_{c,k}$ are the state-space model matrices. Details on the derivation of (1) are given in (Rese et al., 2012a).

2.2 VSI controller

In microgrids operating in grid-connected mode, the VSI controller objective is to regulate the VSI output power to follow preset active and reactive power references, using only local information as inputs. The $k$-th VSI control system state-space equation is given by:

$$
\Delta \dot{x}_k = A_{i,k} \Delta x_k + B_{i,k} \Delta i_{dq,k} + B_{r,k} \Delta i_{c,k}
\Delta y_k = C_{i,k} \Delta x_k + D_{i,k} \Delta i_{dq,k} + D_{c,k} \Delta i_{c,k}
$$

(2)

where $x_{i,k}$ is the controller state vector, $i_{dq,k}$ is the VSI grid-side current vector, $S_{ref,k}$ is the reference input vector, $A_{i,k}$, $B_{i,k}$, $B_{r,k}$, $C_{i,k}$, $D_{i,k}$, $D_{c,k}$ and $D_{r,k}$ are the state-space model matrices. The complete description of the VSI controller model is given in Section 3.

2.3 Network model

Microgrids make widespread use of electronically interfaced microsources. These sources have low or even null inertia, which makes it necessary to take into account some fast dynamic phenomena as, for example, network and VSI filter dynamics.

In this paper, the network state equations, represented in a $dq$ reference frame, are derived according to the same methodology employed by Anderson et al. (1989) and Parmiani and Iravani (1995). It is assumed that every line or transformer can be represented by a series RL branch. The node representing the main grid is represented as an infinite node and is modeled as an ideal three-phase voltage source. For network modeling purposes, microsources are represented as current sources connected to the generation nodes. Loads are represented as series RL or parallel RC circuits, although more complex dynamic load models could also be used.

The network small-signal model is then derived through a systematic procedure based on graph theory (Deo, 1974), which facilitates its computer implementation and also the extension
of the proposed methodology to other microgrid configurations. The resulting network small-signal model is given by:

\[
\Delta x_n = A_n \Delta x_n + B_{n,\mu} \Delta u'_{n,\mu} \\
\Delta y_n = C_n \Delta x_n + D_{n,\mu} \Delta u'_{n,\mu}
\] (3)

where \(x_n\) is the network state vector, \(u'_{n,\mu}\) is the input vector, \(A_n\), \(B_{n,\mu}\), \(C_n\) and \(D_{n,\mu}\) are the state-space model matrices. Details on the derivation of (3) and the resulting state-space model matrices are given in (Rese et al., 2012b).

2.4 Dynamic load model

In this paper, capacitive loads are represented as parallel RC branches connected to the ground. Inductive loads are represented as series RL branches connected to the ground. These load models are included in the network model. However, the developed methodology provides flexibility to include other load models in the form of

\[
\Delta x_{ld} = A_{ld} \Delta x_{ld} + B_{ld} \Delta u_{ld} \\
\Delta y_{ld} = C_{ld} \Delta x_{ld} + D_{ld} \Delta u_{ld}
\] (4)

where \(x_{ld}\) is the load model state vector, \(u_{ld}\) the input vector, \(A_{ld}, B_{ld}, C_{ld}\) and \(D_{ld}\) are the state-space model matrices and \(y_{ld}\) must be the load terminal voltage or its current injection at the point of connection to the network.

2.5 Interfacing network, VSI and VSI controller models

Interfacing VSI, VSI controllers and network state-space models requires that every system component be represented with respect to a common (global) reference frame. The relationship between the \(k\)-th local reference frame and the global reference frame is depicted in Fig. 2.

Figure 2: Local and global reference frames

The relationship between a variable \(f_{k,q,l}^{l,d}\) in local coordinates and its representation in the global reference frame \(f_{k,q,g}^{l,d}\) is given by:

\[
\begin{bmatrix}
  f_{k,q,g}^{l,d} \\
  f_{k,q,g}^{r,g}
\end{bmatrix} = \begin{bmatrix}
  \cos \delta_k & -\sin \delta_k \\
  \sin \delta_k & \cos \delta_k
\end{bmatrix}
\begin{bmatrix}
  f_{k,q,l}^{l,d} \\
  f_{k,q,l}^{r,l}
\end{bmatrix}
\] (5)

where \(\delta_k\) is defined in Fig. 2.

The transformation of each microgrid component model to the global reference frame is realized by applying the linearized version of (5) to the input and output vectors of each microgrid component model (except the network model, that is naturally derived in the global reference frame). Details on the transformation of the individual models to the global reference frame are given in (Rese et al., 2012b).

Figure 3 shows the input/output relationships of a VSI, the VSI controller and the network.

Figure 3: Input/output relationship between VSI, VSI controller and network models

The connection of VSI and VSI controller state-space equations results in the following microsource equation:

\[
\begin{align*}
\Delta x'_{ld,\mu} &= A'_{ld,\mu} \Delta x_{ld,\mu} + B'_{ld,\mu} \Delta v'_{ld,\mu} + B'_{ld,\mu} \Delta S_{ref} \\
\Delta y_{ld,\mu} &= C'_{ld,\mu} \Delta x_{ld,\mu} + D'_{ld,\mu} \Delta v'_{ld,\mu} + D'_{ld,\mu} \Delta S_{ref}
\end{align*}
\] (6)

where:

\[
x_{ld,\mu} = [x_{ld}^T x_{ld}^T]^T
\]

\[
A'_{ld,\mu} = \left[ (A_{ld} + B_{ld} E_{VSI}) N_{vSI}^{-1} C_{vSI} \right]
\]

\[
B'_{ld,\mu} = B_{ld} E_{VSI}
\]

\[
C'_{ld,\mu} = \left[ \left( D_{ld} E_{VSI} \right)^{-1} \right]
\]

\[
D'_{ld,\mu} = \left( D_{ld} E_{VSI} \right)^{-1}
\]

The microgrid small-signal model is then obtained by connecting the microsource and network equations, considering the input/output relationships shown in Fig. 3. The resulting model is given by:

\[
\begin{align*}
\Delta x_{pq,\mu} &= A_{pq,\mu} \Delta x_{pq,\mu} + B_{pq,\mu} \Delta S_{ref} \\
\Delta S_{meas} &= C_{pq,\mu} \Delta x_{pq,\mu} + D_{pq,\mu} \Delta S_{ref}
\end{align*}
\] (7)

where:

\[
x_{pq,\mu} = [x_{pq}^T x_{pq}^T]^T
\]

\[
S_{meas} = [P_{meas} Q_{meas}]^T
\]

\[
A_{pq,\mu} = \left[ \left( A_{pq} + B_{pq} D_{pq}^{-1} C_{pq} \right) \right]
\]

\[
B_{pq,\mu} = B_{pq} E_{pq}
\]

\[
C_{pq,\mu} = \left[ [C_{pq} 0] \right]
\]

\[
D_{pq,\mu} = 0
\]

\[
N = (I - D_{pq,\mu} D_{pq,\mu})^{-1}
\]
3 VSI Controller

The objective of VSI control in microgrids operating in grid-connected mode is to ensure active and reactive power injection levels according to preset values as well as to provide active damping for the LCL filter. In this paper, the VSI controller is implemented in the dq reference frame, which makes the control problem a constant reference tracking problem.

Implementation of VSI controller in dq frame requires local information about grid frequency and phase angle to perform the dq0 transformation and to synchronize the VSI with the grid. This can be accomplished through the use of a Phase-Locked Loop (PLL). The PLL system, implemented in dq reference frame, uses a PI controller to track the phase angle of the grid voltage. The lock is realized by setting the reference of q-axis to zero so that, in steady-state, \( v_q = 0 \) (Chung, 2000).

The control strategy used in this paper is based on the multiloop controller configuration shown in Fig. 4. The current controller (inner loop) is responsible for providing active damping to the VSI’s LCL output filter, as well as to ensure reference current tracking capability. The active/reactive power controllers (outer loop) are responsible for generating the \( d \)- and \( q \)-axis reference currents based on preset active and reactive power injection levels.

![Figure 4: VSI controller structure](image)

Controller inputs are the VSI grid-side current and terminal voltage, as well as preset active and reactive power references. Controller outputs are the reference signals for the PWM modulator.

This control strategy can be divided in three main blocks: measurement filtering; reference current calculation based on active/reactive power references; current controller. A more detailed diagram of the VSI controller proposed in this paper is given in Fig. 5. Detailed description of each control block is given in the sequel.

![Figure 5: VSI controller for grid-connected mode](image)

3.1 Measurement filtering block

The measurement filtering block comprises four first-order low-pass filters (LPF) for the \( dq \) components of VSI output current and terminal voltage. The linearized state equations of the filter module are given by:

\[
\begin{align*}
\Delta v_d &= \frac{1}{T_f} \Delta i_d + \frac{1}{T_f} \Delta v_d \\
\Delta i_d &= \frac{1}{T_f} \Delta i_d + \frac{1}{T_f} \Delta i_d \\
\Delta i_q &= \frac{1}{T_f} \Delta i_q + \frac{1}{T_f} \Delta i_q
\end{align*}
\]

where \( T_f \) is the filter time constant (usually, \( 1ms \leq T_f \leq 4ms \)) and the superscript \( f \) stands for filtered quantities.

3.2 Active/reactive power control loop

In this paper, the convention used for abc to dq0 reference frame transformation assumes that the quadrature axis leads the direct axis, and also maintains the one-to-one correspondence between peak values of voltages and currents in abc and dq0 frames (Kundur, 1994). In consequence, the expressions for active and reactive power injections are given by:

\[
P = \frac{3}{2} (v_d i_d + v_q i_q), \quad Q = \frac{3}{2} (v_d i_q - v_q i_d)
\]

Because in steady-state \( v_q = 0 \), the active and reactive power injection expressions become:

\[
P = \frac{3}{2} v_d i_d, \quad Q = -\frac{3}{2} v_d i_q
\]

According to (10), active and reactive power injection must be controlled through \( d \)- and \( q \)-axis current components, respectively. It should be noticed that a positive reactive power reference results in a negative \( q \)-axis current. As consequence, the reactive power control loop exhibits an inverse response characteristic.

Direct- and quadrature-axis reference currents are obtained through two PI controllers: one for the active power loop and the other for the reactive power control loop. The transfer functions for the active and reactive power PI controllers are given by:

\[
P_{\text{PI}}(s) = K_p^a + \frac{K_i^a}{s}, \quad P_{\text{PI}}(s) = K_p^q + \frac{K_i^q}{s}
\]

The compact form of the power control loop linearized state-space model, including the measurement filtering state equations (8), is given by:

\[
\Delta x_{p,m} = A_{\text{p,m}} \Delta x_{p,m} + B_{\text{p,m}} \Delta v_{d} + B_{\text{p,m}} \Delta v_{q} + B_{\text{p,m}} \Delta i_{d} + B_{\text{p,m}} \Delta i_{q} + B_{\text{p,m}} \Delta S_{s,f}
\]

\begin{align*}
\Delta v_{d} &= [A_{\text{p,m}}] [x_{d}, v_{d}]_{t} + \Delta S_{s,f} \Delta v_{d} \\
\Delta v_{q} &= [A_{\text{p,m}}] [x_{q}, v_{q}]_{t} + \Delta S_{s,f} \Delta v_{q}
\end{align*}

where \( A_{\text{p,m}}, B_{\text{p,m}}, B_{\text{p,m}} \) are state-space model matrices, \( v_{d,q} = [v_d, v_q]_{t} \), \( i_{d,q} = [i_d, i_q]_{t} \), \( S_{s,f} = [P_{\text{ref}}, Q_{\text{ref}}]_{t} \) and \( x_{p,m} = [v_{d}, v_{q}, i_{d}, i_{q}, S_{s,f}]_{t} \).

The detailed expression of (12) is not shown in this paper due to space limitations.
3.3 Current control loop

The current control strategy employed in this paper is based on the state-feedback theory, as already proposed by Rese et al. (2012a). This multivariable control strategy avoids the use of major approximations generally used for decoupling d- and q-axis variables (Green and Prodanovic, 2007).

State-feedback controllers require, in principle, the measurement of all state variables, which turns this approach unfeasible in most real applications. In order to circumvent this problem, a state observer is employed (Chen, 1984), making it possible to estimate all state variables with only measurements of the VSI output current and knowing VSI input control signals.

Another characteristic of state-feedback controllers is that they cannot ensure reference tracking. However, reference tracking capability can be achieved by placing integrators in the forward path in series with the plant inputs. It should be noticed that every placed integrator increases the size of the plant by 1, requiring an augmented plant model, given by:

\[
\begin{bmatrix}
\dot{x}_1 \\
\end{bmatrix} = A \begin{bmatrix}
\dot{x}_1 \\
\end{bmatrix} + B u + \begin{bmatrix}
0 \\
\end{bmatrix} r + \begin{bmatrix}
\dot{r}
\end{bmatrix}
\]

where \(x\) is the plant state vector, \(x_1\) the integrator state vector, \(u\) the plant input vector (references for the PWM modulator), \(r\) the reference input vector, \(A\), \(B\) and \(C\) are the plant state-space model matrices, and \(I\) stands for an identity matrix with appropriate dimensions. The block diagram of the proposed current control strategy is shown in Fig. 6.

![Figure 6: Current controller block diagram. In blue, the state observer. In red, the integral action for current reference tracking. In green, the state feedback.](image)

The state-feedback controller design is then carried out using (13), resulting in the state feedback gain matrix \(K_f\) along with the integrators gain matrix \((K_f)\). The state observer design is carried out using the original plant model (without the integrators), resulting in a gain matrix \(L\). Matrices \(K\), \(K_f\), and \(L\) are obtained through a pole placement procedure (Chen, 1984).

The compact form of the state-feedback based current controller linearized model is given by:

\[
\begin{align*}
\Delta x_{sf} &= A_{sf} \Delta x_{sf} + B_{sf} \Delta \delta_q + B_{c,sf} \Delta \delta_{eq} \\
\Delta y_{sf} &= C_{sf} \Delta x_{sf} + D_{sf} \Delta \delta_q + D_{c,sf} \Delta \delta_{eq}
\end{align*}
\]

where \(x_{sf}\) is the state vector, \(A_{sf}\), \(B_{sf}\), \(B_{c,sf}\), \(C_{sf}\), \(D_{sf}\) and \(D_{c,sf}\) are the state-space model matrices, \(\delta_{r,eq}\) is the current reference input vector and \(\delta_{eq}\) stands for the filtered grid-side current input vector.

3.4 Complete VSI controller model

The final VSI controller state-space model is obtained by connecting equations (12) and (14) according to the input/output relationships shown in Fig. 4. The resulting state-space model is given by:

\[
\begin{align*}
\Delta x_c &= A_c \Delta x_c + B_{c,sf} \Delta \delta_q + B_{c,sf} \Delta \delta_{eq} + B_{c,s} \Delta \delta_{eq} \\
\Delta y_c &= C_c \Delta x_c + D_{c,sf} \Delta \delta_q + D_{c,sf} \Delta \delta_{eq} + D_{c,s} \Delta \delta_{eq}
\end{align*}
\]

where \(x_c\) is \([x_{per}^T \ x_{eq}^T]^T\),

\[
A_c = \begin{bmatrix}
A_{per} \\
B_{c,sf} H + B_{c,sf} C_{per} A_{sf} \\
B_{s,f} D_{per} \\
B_{r,sf} D_{per} \\
C_{c}
\end{bmatrix},
\]

\[
B_c = \begin{bmatrix}
B_{c,sf} \delta_{ref} \delta_{ref} \\
B_{s,f} \delta_{ref} \delta_{ref} \\
B_{pwr} r_{ref} \\
B_{pwr} r_{ref} \\
B_{pwr} r_{ref}
\end{bmatrix},
\]

\[
C_c = \begin{bmatrix}
D_{c,sf} \delta_{ref} \delta_{ref} \\
D_{s,f} \delta_{ref} \delta_{ref} \\
D_{s,f} \delta_{ref} \delta_{ref} \\
D_{pwr} r_{ref} \\
D_{pwr} r_{ref} \\
D_{pwr} r_{ref}
\end{bmatrix},
\]

\[
H = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}.
\]

4 VSI Controller Design

In this paper, the controller design is performed considering VSI local models, that is, assuming that each inverter is directly connected to an infinite node. This is done because, in microgrid applications, the network topology and the number and location of connected microsources and loads are unknown, due to consumer active participation (Chowdhury et al., 2009). Therefore, there is no point in designing these controllers considering preset microgrid configurations.

The VSI controller design procedure is divided into two parts: the current control loop is first designed, and then the active/reactive power controller design is performed.

4.1 Current controller design

Designing the state-feedback current controller requires the specification of the desired closed-loop eigenvalues location. The desired eigenvalues are computed by moving the poor damped modes to the left of the complex plane without changing their imaginary parts in order to satisfy a minimum damping ratio specification. This amounts to computing new values for the eigenvalues’ real parts that meet the specification, as follows:

\[
\lambda_{new} = -\left(\frac{\zeta_m \lambda_{ij}}{\sqrt{1 - (\zeta_m)^2}} \right) + j \Im \left\{ \lambda_{ij} \right\}
\]

where \(\lambda_{new}^{\text{new}}\) corresponds to the desired closed-loop eigenvalues, \(\zeta_m\) is the specified minimum damping ratio and \(\Im \left\{ \lambda_{ij} \right\}\) stands for the imaginary part.
of $\lambda_{ij}$. The desired poles of the observer are defined as $2\text{Re}\{\lambda^{\text{new}}\} + j\text{Im}\{\lambda^{\text{new}}\}$. The design of the controller and the observer is then carried out through a pole placement algorithm.

4.2 Active/reactive power controller design

The design of PI controllers for active and reactive power control loops is based on the following transfer functions:

- Active power controller: $G_a(s) = \frac{P_{\text{max}}(s)}{I_{q}^{\text{ref}}(s)}$.
- Reactive power controller: $G_q(s) = \frac{Q_{\text{max}}(s)}{I_{q}^{\text{ref}}(s)}$.

The controller design procedure is then carried out by solving the following nonlinear programming problem:

$$\min \quad f(x)$$

s.t.  \quad $C(x) \leq (1 - \omega)\gamma$

$$\mathcal{K} \leq x \leq \mathcal{X}$$

$$\gamma \geq 0$$

where $x$ is the optimization variables vector, $f(x)$ is the reference tracking error objective function, $C(x)$ represents equality and inequality constraints (such as limits on damping ratio, rising time, settling time, overshoot, etc.), $\omega$ is a weighting factor and $\gamma$ is the slack variables vector. The constraints are piecewise linear bounds, represented as

$$y_1(t) \quad t_1 \leq t \leq t_2$$

$$\vdots$$

$$y_n(t) \quad t_n \leq t \leq t_{n+1}$$

and are formulated based on the maximum signed distance between lower and upper bounds, as follows:

$$C_l = \begin{bmatrix}
\max_{t_1 \leq t \leq t_2} y_{\text{sim}} - y_{\text{meas}} \\
\vdots \\
\max_{t_n \leq t \leq t_{n+1}} y_{\text{sim}} - y_{\text{meas}}
\end{bmatrix}, \quad C_u = \begin{bmatrix}
\max_{t_1 \leq t \leq t_2} y_{\text{meas}} - y_{\text{sim}} \\
\vdots \\
\max_{t_n \leq t \leq t_{n+1}} y_{\text{meas}} - y_{\text{sim}}
\end{bmatrix}$$

where $y_{\text{sim}}$ is the simulated plant output response (Mat, 2010). The optimization problem is then solved through an interior point algorithm.

5 Small-Signal Stability Analysis and Time-Domain Simulation

The microgrid test system used in this paper is based on the CERTS Microgrid Test Bed (Lasseter, 2009). This microgrid is composed of 8 nodes, 2 microsources and 4 inductive loads, as shown in Fig. 7. Node 1 represents the point of common coupling (PCC) of the microgrid to the utility system. System operating point, network and VSI parameters are given in (Rese et al., 2012b).

Results of the VSI controller design procedure described in Section 4 applied to the microgrid test system are presented in the sequel. It has to be noticed that only VSI-4 controller design is presented. However, a similar procedure is employed for VSI-7.

5.1 VSI controller design

The minimum damping ratio specified for the current control loop design is 35%. Gain matrices $K$, $K_I$ and $L$ are obtained through a pole placement algorithm implemented in Matlab and are not presented in this paper due to space limitations.

Once the state-feedback and observer gain matrices are known, it is then necessary to specify initial values for the active/reactive power PI controllers parameters. In this paper, these initial values are chosen as presented in Table 1.

Table 1: Initial values for active/reactive power PI controller parameters

<table>
<thead>
<tr>
<th>VSI</th>
<th>$K_2^a</th>
<th>K_2^q</th>
<th>K_2^a</th>
<th>K_2^q</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 and 7</td>
<td>0.001</td>
<td>0.010</td>
<td>0.001</td>
<td>0.010</td>
</tr>
</tbody>
</table>

The constraints for the PI controllers design are the following:

- Minimum damping ratio: 20%;
- Rise time (80%): 0.3 seconds;
- Settling time (100%): 0.6 seconds;
- Overshoot: 10%;
- Undershoot: 1%.

Figure 8 shows the root locus with the minimum damping ratio specification for the VSI-4 controller considering the initial controller parameters values. Figure 9 shows the step response for the VSI-4 considering the initial controller parameters values, as well as the step response constraints for controller design purposes. It can be seen that, despite the fact that the minimum damping ratio condition is satisfied with the parameters’ initial values, the step response constraints are not satisfied.

The resulting root-locus and step responses for the VSI-4 controller designed according to the methodology described in Section 4 are given in Fig. 10 and 11, respectively. It can be seen that the tuned controller satisfies all controller design specifications. Final parameters for both VSI-4 and VSI-7 controllers are given in Table 2.
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reason for the negative integral gain in reactive power PI controllers is the inverse response of the reactive power loop, as explained in Subsection 3.2.

5.3 Nonlinear time-domain simulations

Time-domain simulations of the microgrid system shown in Fig. 7, including the VSI controllers, has been performed using the PSCAD/EMTDC software package. These simulations are used to validate the results obtained through the microgrid eigenanalysis and also to check the performance of the proposed VSI controller. It should be emphasized that the simulations are carried out using nonlinear models for the PLL and the microgrid, as well as the switching model of the VSIs, thus enabling the representation of the non-modeled dynamics excited by the switching actions of the VSIs.

Figure 12 shows VSI-4 responses of active/reactive power injection, d- and q-axis components of grid-side current and grid-side currents in abc frame for the following events: at t = 200 ms, active power reference is changed from 0 kW to 20.0 kW, while the reactive power reference is kept at 0. At t = 550 ms, active and reactive power references are changed to 30 kW and 5 kVA, respectively.

Figure 13 shows VSI-7 responses of active/reactive power injection, d- and q-axis components of grid-side current and grid-side currents.

![Figure 8: VSI-4 root locus with initial values for controller parameters and desired damping ratio region](image)

![Figure 9: VSI-4 step with initial values for controller parameters and step response constraints](image)

![Figure 10: VSI-4 root locus with tuned controller parameters and desired damping ratio region](image)

![Figure 11: VSI-4 step with tuned controller parameters and step response constraints](image)

![Figure 12: VSI 4 simulation results](image)

![Figure 13: VSI-7 responses of active/reactive power injection, d- and q-axis components of grid-side current and grid-side currents](image)
in abc frame for the following events: $t = 300 \text{ ms}$ active and reactive power references are changed from 0 kW and 0 kVAr to 40.0 kW and 10 kVAr, respectively. At $t = 550 \text{ ms}$, active and reactive power references are changed to 50 kW and 15 kVAr, respectively.

Figure 13: VSI 7 simulation results

It can be noticed that, in both cases, the proposed VSI control strategy proved to be a good alternative for VSIs operating in grid-connected mode. It is also possible to validate the results obtained through the eigenanalysis presented in Subsection 5.2.

6 Conclusion

The mathematical formulation of a small-signal model of microgrids operating in grid-connected mode, including the dynamics of VSIs with LCL output filters and network representation, has also been proposed. This model is suitable for the analysis of complex dynamic phenomena that arise with the widespread use of VSIs in microgrid applications.

A VSI controller structure for operation in grid-connected mode, as well as the controller design procedure, has been presented. This controller has proven to be able to meet the specifications for the connection of the VSI to microgrids operating in grid-connected mode. The validation of the proposed VSI control strategy has been carried out through small-signal stability analysis using the developed model, as well as time-domain nonlinear simulations via PSCAD/EMTDC software package.

It must be emphasized that the developed control structure is intended to application only in grid-connected operating mode. The VSI control problem for islanded operating mode will be addressed in future research work.

References


