Robust $H_\infty$ Control for Grid Connected PWM Inverters with LCL Filters

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Abstract—This paper provides a robust control design based on linear matrix inequalities applied to PWM inverters with LCL filters connected to the grid under uncertain parameters. The proposed controller ensures: i) closed-loop robust stability against filters connected to the grid under uncertain parameters. The linear matrix inequalities applied to PWM inverters with LCL system, as well as limits of robust stability. Simulation and experimental results show good transient and frequency responses for the closed-loop system, as well as limits of robust stability.

I. INTRODUCTION

Renewable energy sources have become progressively more important, being PWM inverters a key stage used as an interface between renewable energy sources and the power grid [1]. LCL filters are often used in the output of PWM inverters to attenuate harmonics, providing suitable current to be injected to the grid, allowing the control of the power flow between the primary source and the grid. In this context, one important problem is the attenuation of the high peak of resonance of the third order LCL filter, which can be taken into account by a suitable controller [2], [3], [4], [5], [6], [7].

Concerning control techniques, current control for grid-connected converters with LCL filters can be found in literature using proportional plus integral (PI) controller in rotating frames or proportional plus resonant (PR) controllers in stationary frame. Both techniques are capable to cope with the problem of resonance of LCL filter and with rejection of disturbances in the current injected into the grid. Rotating synchronous frame PI controllers are used in three-phase inverters to obtain reference tracking with zero steady-state error, as in [9], where they were used jointly with auxiliary filters which deal with the resonance damping. In [10], the resonance damping is carried out with the help of an extra feedback loop. Proportional-resonant controllers are designed in stationary frame introducing an infinite gain in specific frequencies to eliminate steady-state error and to reject specific harmonic disturbances, being applied to photovoltaic inverters in [11], and in [12] where resonant controllers were designed with a new method that takes into account one extra current sensor to improve the harmonic rejection. However, the control of the current injected to the grid is not an easy task since, in practice, the system can be affected, for instance, by uncertainties in the grid impedance and by grid voltage disturbances. This leads to the interest in robust control techniques as [13], [14], [15]. In [14], a robust controller has been successfully implemented using a partial state feedback to ensure robustness to uncertainties in the grid and then, based on a heuristic procedure in the frequency domain, a set of resonant controllers was designed. This work indicates the viability for this application of robust controllers based on linear matrix inequalities (LMIs, [16]). More recently, in [15] robust pole location was used for a more general set of resonant controllers in the state space formulation, leading to good results. However, optimal disturbance rejection was not investigated in these works and the $H_\infty$ framework is appealing for studies.

The $H_\infty$ control can provide optimal rejection of disturbances. In the LMI formulation, the problem is easily solved, allowing to include other specifications as pole location [17], [18], [19], [16], [20], [21], [22]. It is known that $H_\infty$ controllers frequently leads to high control gains, which can be difficult to be implemented in practice. For these controllers, tradeoffs between optimality and performance must be investigated. In the sequence, this paper addresses the investigation of viability of robust $H_\infty$ controllers for grid connected converters with LCL filters. First, a polytopic model of the system is provided. The state feedback controller proposed has two actions: the feedback of the converter states, which receives the vector gain $K_1$, and the feedback of the resonant controller states, which receives the vector gain $K_2$. Differently from [14], these gains are computed all simultaneously, in an efficient design procedure, based only in the choice of two parameters for pole location and also on a prescribed $H_\infty$ guaranteed cost. Tradeoffs between performance and size of control gains can be investigated. Better performance than that from simple pole location can be produced [15], even with a simpler resonant controller than those in [14], [15]. Good simulation and experimental results are provided by the proposed controller under uncertainties on grid parameters.
II. MODELING OF GRID-CONNECTED VOLTAGE SOURCE CONVERTERS WITH LCL FILTER

Consider the single-phase converter presented in Fig. 1. A continuous-time state space linear model is given by

$$\frac{d}{dt} \begin{bmatrix} i_e \\ v_c \\ i_g \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L_e} & 0 \\ \frac{1}{C} & 0 & -\frac{1}{L} \\ 0 & \frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} i_e \\ v_c \\ i_g \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{C} \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{L} \end{bmatrix} v_d$$

The grid inductance parameter is modeled as

$$L_g = L_{g1} + L_{g2}$$

The states are the converter current, $i_e$, the capacitor voltage, $v_c$, and the grid current, $i_g$. The input $u$ is the voltage applied by the converter, the output is the grid current, $i_g$, and the disturbance is the grid voltage, $v_d$.

Rewriting (1) in a compact form, one has

$$\dot{x}(t) = A_e x(t) + B_e u(t) + B_{dc} v_d(t)$$
$$y(t) = C_e x(t) = i_g(t)$$

A discrete-time representation of (3), with sampling period $T_s$, is given by

$$x(k+1) = e^{A_e T_s} x(k) + \sum_{\tau=0}^{T_s} e^{A_e (T_s-\tau)} B_{dc} u(k)$$
$$+ \sum_{\tau=0}^{T_s} e^{A_e (T_s-\tau)} B_{dc} v_d(k)$$

Including an one sample delay from the digital implementation of the control, one has

$$\begin{bmatrix} x(k+1) \\ \phi(k+1) \end{bmatrix} = \begin{bmatrix} G & H \\ 0_{1\times3} & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ \phi(k) \end{bmatrix} + \begin{bmatrix} 0_{3\times1} & 0 \\ 0 & 1 \end{bmatrix} u(k) + \begin{bmatrix} H_d \\ 0 \end{bmatrix} v_d(k)$$

An alternative for tracking sinusoidal references and for rejection of disturbances is to use a resonant controller with frequency $\omega_0$, which can be written in the discrete-time state space form, as

$$\xi(k+1) = R \xi(k) - TC_c x(k) + T i_{ref}(k)$$

where $\xi \in R^{2\times1}$. Taking into account that the tracking error is given by $e(k) = i_{ref}(k) - C_c x(k)$, then, from (5) and (6), the augmented system can be expressed as

$$\begin{bmatrix} x(k+1) \\ \phi(k+1) \\ \xi(k+1) \end{bmatrix} = \begin{bmatrix} G & H & 0_{3\times2} \\ 0_{1\times3} & 0 & 0_{1\times2} \\ -TC_c & 0 & R \end{bmatrix} \begin{bmatrix} x(k) \\ \phi(k) \\ \xi(k) \end{bmatrix} + \begin{bmatrix} 0_{3\times1} \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} H_d \\ 0 \end{bmatrix} v_d(k) + \begin{bmatrix} 0_{3\times1} \\ 0 \end{bmatrix} i_{ref}(k)$$

or, in a compact form

$$\rho(k+1) = A \rho(k) + B u(k) + B_r i_{ref}(k) + B_d v_d(k)$$

$$y(k) = C \rho(k)$$

where $\rho \in R^{6\times1}$, $u \in R$ and $y \in R$ with matrices $A \in R^{6\times6}$, $B \in R^{6\times1}$, $B_r \in R^{6\times1}$, $B_d \in R^{6\times1}$ and $C \in R^{1\times6}$.

Assuming the state feedback control law

$$u(k) = K \rho(k)$$

with $K = [K_1 \ K_2]$, one can represent the closed-loop system as shown in Fig. 2. Notice that $K_1$ is related to the states of the plant and with the state concerning the control delay and $K_2$ is related to the state of the resonant controller.

Due to the uncertainty affecting the grid inductance $L_{g2}$ in practical applications, this parameter is considered here belonging to a given interval. Then, (8) can be rewritten as a polytopic model described by [16]

$$\rho(k+1) = A(\alpha) \rho(k) + B(\alpha) u(k) + B_r(\alpha) i_{ref}(k) + B_d(\alpha) v_d(k)$$

where

$$A(\alpha) = \sum_{i=1}^{2} \alpha_i (A, B, B_r, B_d)_i,$$

$$\sum_{i=1}^{2} \alpha_i = 1, \quad \alpha_i \geq 0, \quad i = 1, 2$$
The vertices of the polytopic model in (11) are given by the evaluation of matrices $\{A, B, B_i, B_{di}\}$ for the minimum and maximum values of $L_{g2}$.

### III. ROBUST CONTROL DESIGN WITH LMIs

In order to cope with uncertainties affecting the parameter $L_{g2}$, a robust control design technique based on LMIs is used as shown in the next theorem.

**Theorem 1**: For given real positive scalars $r, d$, defining the circle belonging to the unit circle as shown in Fig.3, and for a given $\mu > 0$, if there exist symmetric positive definite matrices $\mathcal{W}_i \in R^{6 \times 6}$, and matrices $\mathcal{G} \in R^{6 \times 6}$ and $\mathcal{Z} \in R^{1 \times 6}$ such that, for $i = 1, 2$ and $j = 1, 2$

$$
\begin{bmatrix}
\mathcal{G} + \mathcal{G}' - \mathcal{W}_i & * & * & * \\
0 & \mu I & * & * \\
r^{-1}(A_i - dI)\mathcal{G} + r^{-1}\mathcal{B}_i\mathcal{Z} & \mathcal{B}_{di} & \mathcal{W}_j & * \\
C_i\mathcal{G} & 0 & 0 & \mu I
\end{bmatrix} > 0
$$

(12)

hold, then the state feedback gain

$$
K = Z\mathcal{G}^{-1}
$$

(13)

ensures

i) closed-loop robust stability of (10);

ii) that the closed-loop eigenvalues are inside the circle belonging to the unit circle shown in Fig.3 and also that the closed-loop has a prescribed $\mathcal{H}_\infty$ guaranteed cost given by $\mu$;

iii) the tracking of sinusoidal reference and disturbance rejection at frequency $\omega_o$.

**Proof:**

The proof comes directly from Theorem 4 in [23] replacing $A_i$ by $(A_i - dI)/r$ and $B_i$ by $B_i/r$. This ensures properties i) and ii). Property iii) is ensured by the internal model principle control [24], which ensures that a resonant controller as (6) guarantees tracking of sinusoidal reference $i_{ref}(k)$ and rejection of disturbance $v_d(k)$ with frequency $\omega_o$.

Theorem 1 provides the LMIs for the design of a $\mathcal{H}_\infty$ robust control gain. The proper choices of $r$ and $d$ allow to improve the transient responses and proper choice of $\mu$ allows to get sufficiently small control gains to avoid control saturation.

For a design example, consider the parameters of the plant given in Table I. The resonant controller is chosen as

### TABLE I

PARAMETERS OF THE CONVERTER.

<table>
<thead>
<tr>
<th>System description</th>
<th>Nominal values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>3kW</td>
</tr>
<tr>
<td>Grid phase voltage</td>
<td>220V</td>
</tr>
<tr>
<td>Nominal current</td>
<td>13.63A</td>
</tr>
<tr>
<td>Converter inductance $L_c$</td>
<td>1mH</td>
</tr>
<tr>
<td>LCL-filter Grid-side inductance $L_{g1}$</td>
<td>0.5mH</td>
</tr>
<tr>
<td>Filter capacitor $C$</td>
<td>25$\mu$F</td>
</tr>
<tr>
<td>Grid impedance</td>
<td>Max. grid inductance $L_{g2}$</td>
</tr>
<tr>
<td></td>
<td>Min. grid inductance $L_{g2}$</td>
</tr>
</tbody>
</table>

$$
G_c(s) = \frac{s}{s^2 + 2\zeta \omega_o s + \omega_o^2}
$$

(14)

where $\zeta$ is a damping factor employed to avoid problems in the discrete-time implementation related with placing controller poles at the border of the unit circle, and $\omega_o$ is the fundamental frequency. Discretizing model (14) using, for instance, Tustin discretization with sample frequency $1/T_s = 20040$ Hz, $\omega_o = 2\pi \times 60$ rad/s and $\xi = 0.00001$, one has that the state space representation (6) for the resonant controller.

Evaluating the augmented model (7) for the minimum and maximum values of the uncertain parameter $L_{g2}$, for the above sample period $T_s$, one has the vertices of simplified polytopic model given in the Appendix. Applying Theorem 1 for these vertices with the choices $r = 0.49$, $d = 0.5$ and $\mu = 100$, one gets the control gains

$$
K = \begin{bmatrix} K_1 & K_2 \end{bmatrix}
$$

(15)

which lead to the results presented in the next section.

It is important to notice that the design procedure given in this section is different of [14] and [25] because here one has taken into account the $\mathcal{H}_\infty$ performance in the LMIs and only one resonant controller (on the fundamental frequency). Another important difference is that the resonant controller gains $K_2$ in (9) are designed simultaneously with the gains $K_1$ in (9). In [14], first one designs the gains $K_1$ and after, in an heuristic frequency domain procedure designs the gains from the resonant controller. This procedure can demand more time for design of gains for resonant controllers with higher dimension. Finally, one has that the controller proposed here can be easily used for resonant controllers of arbitrary...
dimension by using a set of controller as (14) tuned on different frequencies.

IV. SIMULATION AND EXPERIMENTAL RESULTS

First, to show that the controller ensures the robust stability of the closed-loop system for uncertainties in grid inductance $L_{g2}$, the closed-loop poles for the values of $L_{g2}$ in the interval in Table I are presented in Fig. 4, confirming the robust stability of the closed-loop system.

In order to show the good tracking of reference of the closed-loop system for uncertainties of $L_{g2}$, the Bode diagrams from the reference ($i_{ref}$) to the output current ($i_g$) are shown in Fig. 5 for $L_{g2}$ in the range of Table I. Notice that for 60 Hz, the gains are 0 dB and phase are 0 degrees, as expected.

The good disturbance rejection characteristics of the system with the $H_{\infty}$ controller, specially the rejection of disturbances in the fundamental frequency, is shown in Fig.6, where one can see the Bode diagrams from the disturbance in $v_d$ to the output current ($i_g$) in the range of $L_{g2}$ in Table I.

Fig.7 and Fig.8 present the experimental results for a short circuit test (i.e. $v_d = 0$). One can see the good results in terms of both transient and steady state responses of the $H_{\infty}$ controller for the extreme values of $L_{g2}$ specified in the controller design, indicating the robustness against uncertainties on this parameter. These experimental results have been with a 32-bit floating-point DSP, show the good capacity of tracking for a sinusoidal reference signal with some sudden changes of pattern.

Finally, Fig.9 shows the closed-loop poles for values of $L_{g2}$...
from 0 mH to 25 mH, to show the limits of stability of the closed-loop system Fig.10 is a detailed view of Fig.9 showing some of the poles outside of the unit circle, for values of $L_{g2}$ greater than 17 mH, which points out a large limit of stability for the closed-loop system.

V. CONCLUSION

This paper provided the design and implementation of a robust $\mathcal{H}_\infty$ state feedback controller for grid connected PWM inverters with LCL filters under uncertain parameters. First, a discrete-time augmented model is given, with the inclusion of control delay and of a resonant controller which ensures good reference tracking and disturbance rejection. This resonant controller is used only in the fundamental frequency, for sake of simplicity, but the approach encompasses resonant controllers of arbitrary dimension. A robust $\mathcal{H}_\infty$ controller is designed to cope with uncertainties on the grid parameters and also to ensure control gains suitable for digital implementation. The closed-loop system was tested in simulation, presenting good results as shown by the frequency responses and the closed-loop poles. The experimental results corroborate the good performance for the uncertainties in grid inductance under consideration.

VI. ACKNOWLEDGEMENTS

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REFERENCES


Appendix

Vertex 1, for $L_{g2} = 0$ mH

\[ A_1 = \begin{bmatrix} 0.9514 & -0.0474 & 0.0485 & 0.0490 & 0 & 0 \\ 0.9890 & 0.8542 & -1.8980 & 0.0485 & 0 & 0 \\ 0.0971 & 0.0045 & 0.0028 & 0.0016 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.0078 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 & 0 \end{bmatrix} \]

\[ B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_{d1} = \begin{bmatrix} -0.0016 \\ -0.0071 \\ -0.0965 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \]

\[ C_1 = [0 \ 0 \ 1 \ 0 \ 0 \ 0] \]

Vertex 2, for $L_{g2} = 1$ mH

\[ A_2 = \begin{bmatrix} 0.9508 & -0.0485 & 0.0491 & 0.0490 & 0 & 0 \\ 1.9412 & 0.9181 & -1.9412 & 0.0491 & 0 & 0 \\ 0.0327 & 0.0323 & 0.9672 & 0.0005 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0078 & 0 & 1.9996 & -0.9999 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_{d2} = \begin{bmatrix} -0.0005 \\ 0.0327 \\ -0.0329 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \]

\[ C_2 = [0 \ 0 \ 1 \ 0 \ 0 \ 0] \]