Coordenadas cartesianas

Deslocamento infinitesimal $d\vec{r} = dx\,\hat{\imath} + dy\,\hat{\jmath} + dz\,\hat{k}$

Volume infinitesimal $dV = dx\,dy\,dz$

Gradiente $\nabla\Phi = \dfrac{\partial\Phi}{\partial x}\,\hat{\imath} + \dfrac{\partial\Phi}{\partial y}\,\hat{\jmath} + \dfrac{\partial\Phi}{\partial z}\,\hat{k}$

Divergência $\nabla.\vec{A} = \dfrac{\partial A_x}{\partial x} + \dfrac{\partial A_y}{\partial y} + \dfrac{\partial A_z}{\partial z}$

Rotacional $\nabla\wedge\vec{A} = \left(\dfrac{\partial A_z}{\partial y} - \dfrac{\partial A_y}{\partial z}\right)\hat{\imath} + \left(\dfrac{\partial A_x}{\partial z} - \dfrac{\partial A_z}{\partial x}\right)\hat{\jmath} + \left(\dfrac{\partial A_y}{\partial x} - \dfrac{\partial A_x}{\partial y}\right)\hat{k}$

Laplaciano $\nabla^2\Phi = \dfrac{\partial^2\Phi}{\partial x^2} + \dfrac{\partial^2\Phi}{\partial y^2} + \dfrac{\partial^2\Phi}{\partial z^2}$

Coordenadas cilíndricas

Deslocamento infinitesimal $d\vec{r} = dr\,\hat{\boldsymbol{e}}_r + r\,d\phi\,\hat{\boldsymbol{e}}_\phi + dz\,\hat{\boldsymbol{e}}_z$

Volume infinitesimal $dV = r\,dr\,d\phi\,dz$

Gradiente $\nabla\Phi = \dfrac{\partial\Phi}{\partial r}\,\hat{\boldsymbol{e}}_r + \dfrac{1}{r}\dfrac{\partial\Phi}{\partial\phi}\,\hat{\boldsymbol{e}}_\phi + \dfrac{\partial\Phi}{\partial z}\,\hat{\boldsymbol{e}}_z$

Divergência $\nabla.\vec{A} = \dfrac{1}{r}\dfrac{\partial}{\partial r}(rA_r) + \dfrac{1}{r}\dfrac{\partial A_\phi}{\partial\phi} + \dfrac{\partial A_z}{\partial z}$

Rotacional $\nabla\wedge\vec{A} = \left[\dfrac{1}{r}\dfrac{\partial A_z}{\partial\phi} - \dfrac{\partial A_\phi}{\partial z}\right]\hat{\boldsymbol{e}}_r + \left[\dfrac{\partial A_r}{\partial z} - \dfrac{\partial A_z}{\partial r}\right]\hat{\boldsymbol{e}}_\phi + \dfrac{1}{r}\left[\dfrac{\partial}{\partial r}(rA_\phi) - \dfrac{\partial A_r}{\partial\phi}\right]\hat{\boldsymbol{e}}_z$

Laplaciano $\nabla^2\Phi = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial\Phi}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2\Phi}{\partial\phi^2} + \dfrac{\partial^2\Phi}{\partial z^2}$

Coordenadas esféricas

Deslocamento infinitesimal $d\vec{r} = dr\,\hat{\boldsymbol{e}}_r + r\,d\theta\,\hat{\boldsymbol{e}}_\theta + r\,\sin\theta\,d\phi\,\hat{\boldsymbol{e}}_\phi$

Volume infinitesimal $dV = r^2\,\sin\theta\,dr\,d\theta\,d\phi$

Gradiente $\nabla\Phi = \dfrac{\partial\Phi}{\partial r}\,\hat{\boldsymbol{e}}_r + \dfrac{1}{r}\dfrac{\partial\Phi}{\partial\theta}\,\hat{\boldsymbol{e}}_\theta + \dfrac{1}{r\sin\theta}\dfrac{\partial\Phi}{\partial\phi}\,\hat{\boldsymbol{e}}_\phi$

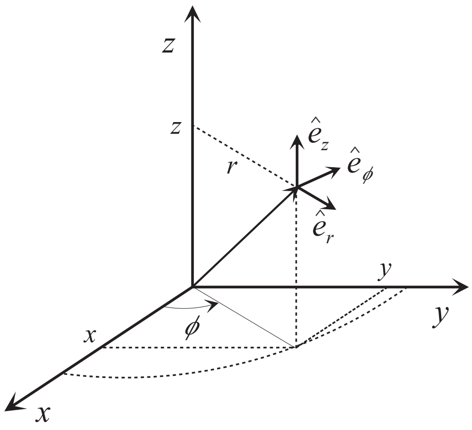
Divergência $\nabla.\vec{A} = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}(r^2A_r) + \dfrac{1}{r\sin\theta}\dfrac{\partial}{\partial\theta}(\sin\theta A_\theta) + \dfrac{1}{r\sin\theta}\dfrac{\partial A_\phi}{\partial\phi}$

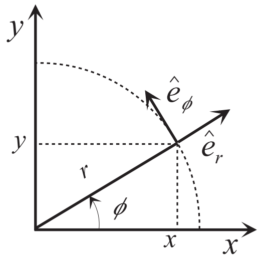
Rotacional $\nabla\wedge\vec{A} = \dfrac{1}{r\sin\theta}\left[\dfrac{\partial}{\partial\theta}(\sin\theta A_\phi) - \dfrac{\partial A_\theta}{\partial\phi}\right]\hat{\boldsymbol{e}}_r + \dfrac{1}{r}\left[\dfrac{1}{\sin\theta}\dfrac{\partial A_r}{\partial\phi} - \dfrac{\partial}{\partial r}(rA_\phi)\right]\hat{\boldsymbol{e}}_\theta +$

$+\dfrac{1}{r}\left[\dfrac{\partial}{\partial r}(rA_\theta) - \dfrac{\partial A_r}{\partial\theta}\right]\hat{\boldsymbol{e}}_\phi$

Laplaciano $\nabla^2\Phi = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial\Phi}{\partial r}\right) + \dfrac{1}{r^2\sin\theta}\dfrac{\partial}{\partial\theta}\left(\sin\theta\dfrac{\partial\Phi}{\partial\theta}\right) + \dfrac{1}{r^2\sin^2\theta}\dfrac{\partial^2\Phi}{\partial\phi^2}$

Coordenadas cilíndricas





$\hat{\boldsymbol{e}}_r \wedge \hat{\boldsymbol{e}}_\phi = \hat{\boldsymbol{e}}_z \qquad \hat{\boldsymbol{e}}_\phi \wedge \hat{\boldsymbol{e}}_z = \hat{\boldsymbol{e}}_r \qquad \hat{\boldsymbol{e}}_z \wedge \hat{\boldsymbol{e}}_r = \hat{\boldsymbol{e}}_\phi$

$x = r\cos\phi \qquad r = \sqrt{x^2+y^2}$

$y = r\sin\phi \qquad \phi = \arctan\dfrac{y}{x}$

$z = z$

$\hat{\boldsymbol{e}}_r = \cos\phi\,\hat{\imath} + \sin\phi\,\hat{\jmath} \qquad \hat{\imath} = \dfrac{x}{\sqrt{x^2+y^2}}\,\hat{\boldsymbol{e}}_r - \dfrac{y}{\sqrt{x^2+y^2}}\,\hat{\boldsymbol{e}}_\phi$

$\hat{\boldsymbol{e}}_\phi = -\sin\phi\,\hat{\imath} + \cos\phi\,\hat{\jmath} \qquad \hat{\jmath} = \dfrac{y}{\sqrt{x^2+y^2}}\,\hat{\boldsymbol{e}}_r + \dfrac{x}{\sqrt{x^2+y^2}}\,\hat{\boldsymbol{e}}_\phi$

$$\frac{\partial \hat{\boldsymbol{e}}_r}{\partial \phi} = \hat{\boldsymbol{e}}_\phi$$

$$\frac{\partial \hat{\boldsymbol{e}}_\phi}{\partial r} = -\hat{\boldsymbol{e}}_r$$

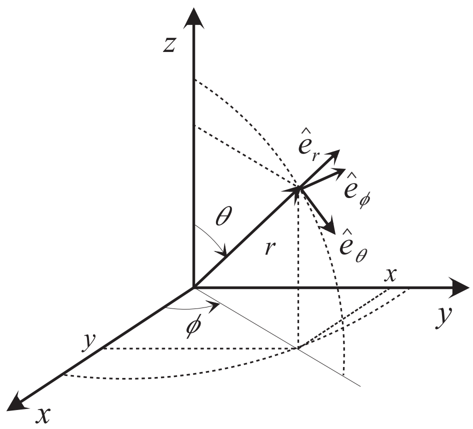
$$\vec{A} = A_x\,\hat{\imath} + A_y\,\hat{\jmath} + A_z\,\hat{k} = A_r\,\hat{\boldsymbol{e}}_r + A_\phi\,\hat{\boldsymbol{e}}_\phi + A_z\,\hat{\boldsymbol{e}}_z$$

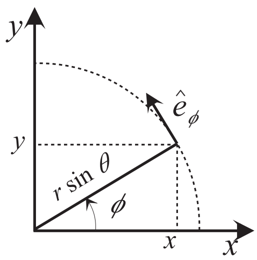
$$A_r = \frac{1}{\sqrt{x^2+y^2}}(xA_x + yA_y)$$

$$A_\phi = \frac{1}{\sqrt{x^2+y^2}}(-yA_x + xA_y)$$

Vector posição $\qquad \vec{r} = r\,\hat{\boldsymbol{e}}_r + z\,\hat{\boldsymbol{e}}_z$

Coordenadas esféricas





$\hat{\boldsymbol{e}}_r \wedge \hat{\boldsymbol{e}}_\theta = \hat{\boldsymbol{e}}_\phi \qquad \hat{\boldsymbol{e}}_\theta \wedge \hat{\boldsymbol{e}}_\phi = \hat{\boldsymbol{e}}_r \qquad \hat{\boldsymbol{e}}_\phi \wedge \hat{\boldsymbol{e}}_r = \hat{\boldsymbol{e}}_\theta$

$$x = r\sin\theta\cos\phi \qquad r = \sqrt{x^2+y^2+z^2}$$

$$y = r\sin\theta\sin\phi \qquad \theta = \arccos\frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$z = r\cos\theta \qquad \phi = \arctan\frac{y}{x}$$

$$\hat{\boldsymbol{e}}_r = \frac{\vec{r}}{r} = \sin\theta\cos\phi\,\hat{\imath} + \sin\theta\sin\phi\,\hat{\jmath} + \cos\theta\,\hat{k} \qquad \hat{\imath} = \frac{x}{\sqrt{x^2+y^2+z^2}}\,\hat{\boldsymbol{e}}_r + \frac{z}{\sqrt{x^2+y^2+z^2}}\frac{x}{\sqrt{x^2+y^2}}\,\hat{\boldsymbol{e}}_\theta - \frac{y}{\sqrt{x^2+y^2}}\,\hat{\boldsymbol{e}}_\phi$$

$$\hat{\boldsymbol{e}}_\theta = \cos\theta\cos\phi\,\hat{\imath} + \cos\theta\sin\phi\,\hat{\jmath} - \sin\theta\,\hat{k} \qquad \hat{\jmath} = \frac{y}{\sqrt{x^2+y^2+z^2}}\,\hat{\boldsymbol{e}}_r + \frac{z}{\sqrt{x^2+y^2+z^2}}\frac{y}{\sqrt{x^2+y^2}}\,\hat{\boldsymbol{e}}_\theta + \frac{x}{\sqrt{x^2+y^2}}\,\hat{\boldsymbol{e}}_\phi$$

$$\hat{\boldsymbol{e}}_\phi = -\sin\phi\,\hat{\imath} + \cos\phi\,\hat{\jmath} \qquad \hat{k} = \frac{z}{\sqrt{x^2+y^2+z^2}}\,\hat{\boldsymbol{e}}_r - \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}\,\hat{\boldsymbol{e}}_\theta$$

$$\frac{\partial\hat{\boldsymbol{e}}_r}{\partial\theta} = \hat{\boldsymbol{e}}_\theta \qquad \frac{\partial\hat{\boldsymbol{e}}_r}{\partial\phi} = \sin\theta\,\hat{\boldsymbol{e}}_\phi$$

$$\frac{\partial\hat{\boldsymbol{e}}_\theta}{\partial\theta} = -\hat{\boldsymbol{e}}_r \qquad \frac{\partial\hat{\boldsymbol{e}}_\theta}{\partial\phi} = \cos\theta\,\hat{\boldsymbol{e}}_\phi$$

$$\frac{\partial\hat{\boldsymbol{e}}_\phi}{\partial r} = -\sin\theta\,\hat{\boldsymbol{e}}_r - \cos\theta\,\hat{\boldsymbol{e}}_\theta$$

$$\vec{A} = A_x\,\hat{\imath} + A_y\,\hat{\jmath} + A_z\,\hat{k} = A_r\,\hat{\boldsymbol{e}}_r + A_\theta\,\hat{\boldsymbol{e}}_\theta + A_\phi\,\hat{\boldsymbol{e}}_\phi$$

$$A_r = \frac{1}{\sqrt{x^2+y^2+z^2}}(xA_x + yA_y + zA_z)$$

$$A_\theta = \frac{1}{\sqrt{x^2+y^2+z^2}\sqrt{x^2+y^2}}(xzA_x + yzA_y - (x^2+y^2)A_z)$$

$$A_\phi = \frac{1}{\sqrt{x^2+y^2}}(-yA_x + xA_y)$$

Vector posição $\qquad \vec{r} = r\,\hat{\boldsymbol{e}}_r$