

# TE903 – Comunicação Digital

## Transmissão Digital em Banda Passante

Evelio M. G. Fernández

3 de setembro de 2019

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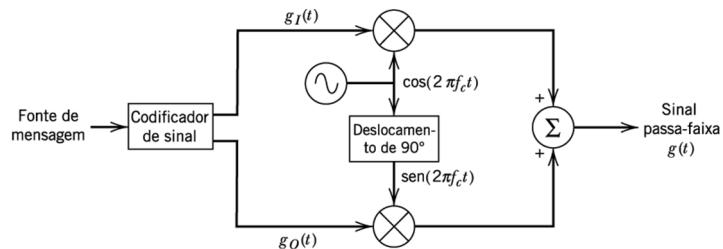
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## Transmissão Passa-Faixa Digital

### Modulador Digital em Quadratura



$$\begin{aligned}g(t) &= g_I(t) \cos(2\pi f_c t) + g_Q(t) \sin(2\pi f_c t) \\&= \Re \left\{ [g_I(t) + jg_Q(t)] e^{j2\pi f_c t} \right\} \\&= a(t) \cos[2\pi f_c(t) + \theta(t)], \quad 0 \leq t \leq T_s\end{aligned}$$

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## Modelo de Canal Gaussiano para Transmissão Passa-Faixa

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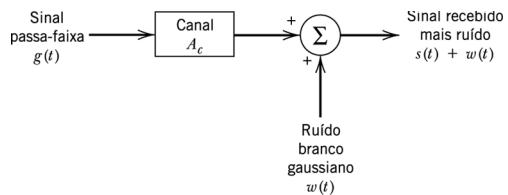
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$$\begin{aligned}x(t) &= s(t) + w(t) \\&= A_c g(t) + w(t), \quad 0 \leq t \leq T_s\end{aligned}$$

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## Demodulador em Quadratura

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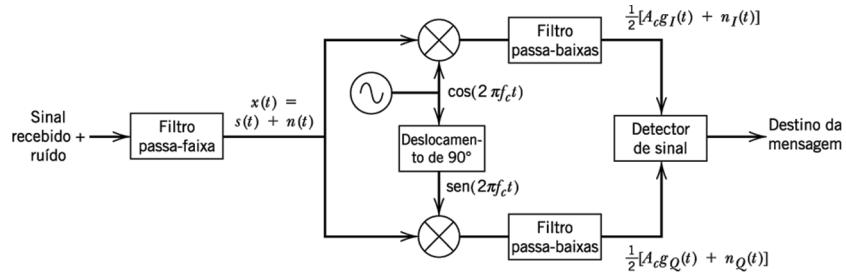
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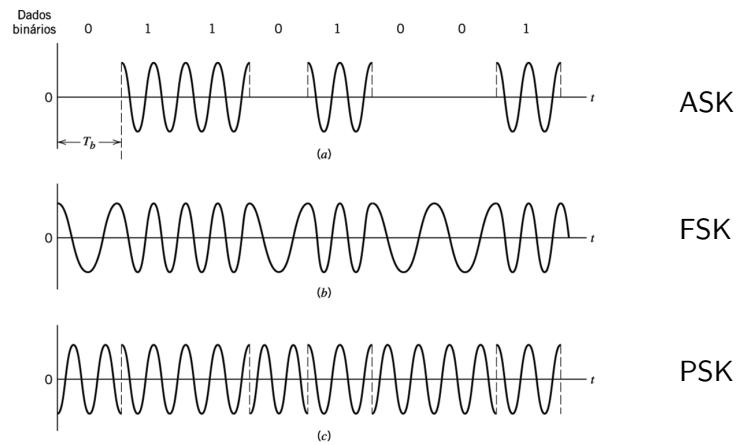
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## Modulações Digitais Básicas



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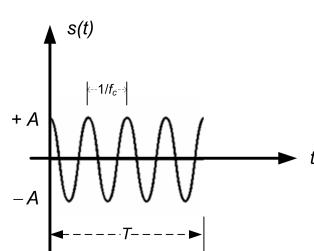
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## Energia de um Pulso de RF



$$E = \int_0^T s^2(t) dt = A^2 \int_0^T \cos^2(2\pi f_c t) dt$$
$$= A^2 \left[ \frac{t}{2} + \frac{\sin(4\pi f_c t)}{4\pi f_c} \right]_0^T$$

$$E = \frac{A^2 T}{2}, \text{ se } f_c = \frac{n}{T}$$

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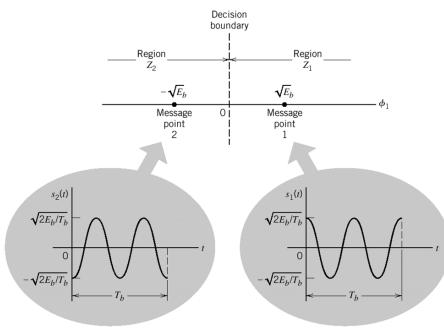
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## Espaço de Sinais – PSK Binário (BPSK)



$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t), \quad 0 \leq t \leq T_b$$

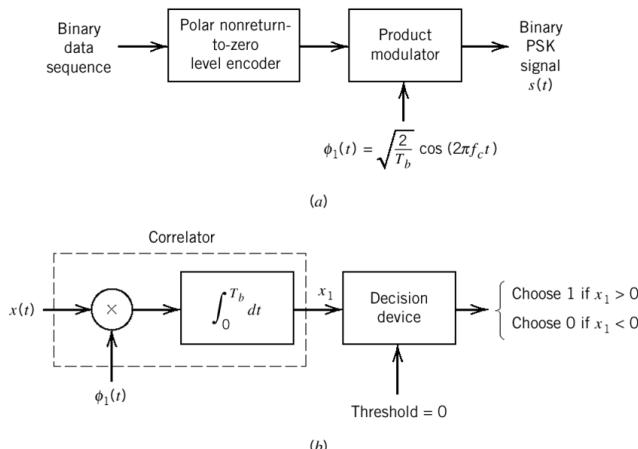
$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi)$$

$$= -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) = -s_1(t)$$

$$\Rightarrow s_1(t) = \sqrt{E_b} \phi_1(t), \quad s_2(t) = -\sqrt{E_b} \phi_1(t)$$

## Geração e Detecção Coerente de Sinais BPSK



$$\text{BER} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}}$$

## Notes

## Notes

## Exercício 1: Problema 6.4 – Haykin

A componente de sinal de um sistema PSK coerente é definida por

$$s(t) = A_c k \sin(2\pi f_c t) \pm A_c \sqrt{1 - k^2} \cos(2\pi f_c t),$$

onde  $0 \leq t \leq T_b$ , o sinal mais corresponde ao símbolo '1' e o sinal menos ao símbolo '0'. O primeiro termo representa um tom piloto a ser utilizado para sincronismo de portadora no receptor.

- Desenhe um diagrama de espaço de sinal para o esquema aqui descrito.
- Mostre que, no canal Gaussiano, a probabilidade média de erro é

$$P_e = \frac{1}{2} \operatorname{erfc} \left( \sqrt{\frac{E_b}{N_0}} (1 - k^2) \right), \text{ onde } E_b = \frac{1}{2} A_c^2 T_b.$$

- Suponha que 10% da potência de sinal transmitida seja alocada à componente de portadora. Determine a  $E_b/N_0$  necessária para se obter uma probabilidade de erro igual a  $10^{-4}$ .
- Compare esse valor de  $E_b/N_0$  com o que seria necessário para um sistema PSK convencional com a mesma probabilidade de erro.

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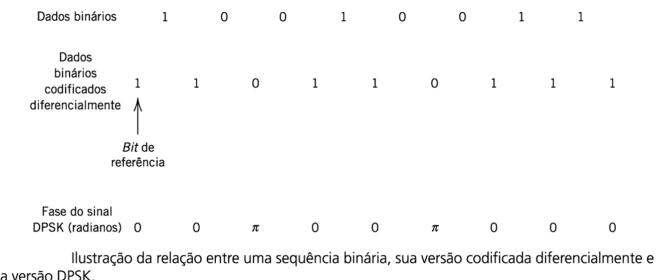
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## Chaveamento de Fase Diferencial



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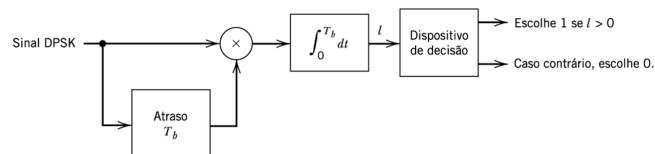
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Receptor para detecção de sinais DPSK.

$$\text{BER} = \frac{1}{2} e^{-\frac{E_b}{N_0}}$$

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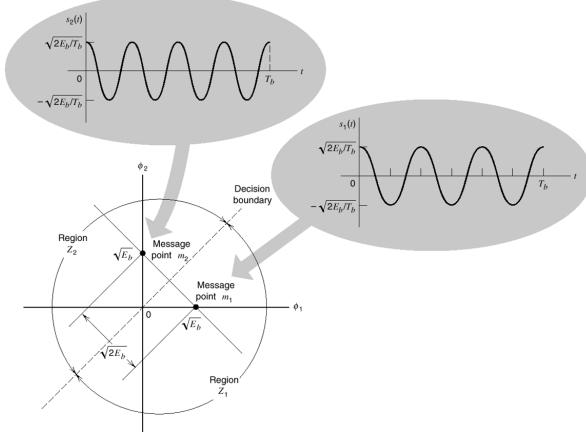
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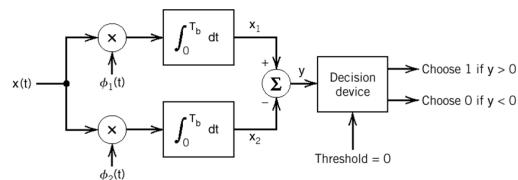
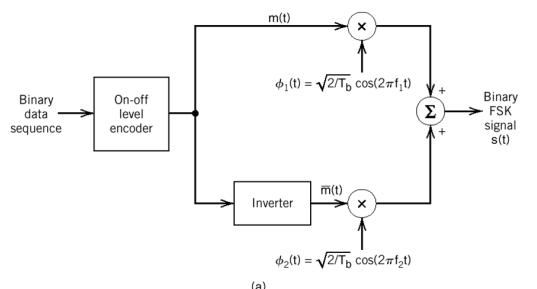
Espaço de Sinais – FSK Binário



$$s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t), & 0 \leq t \leq T_b \\ 0, & \text{fora} \end{cases} \quad \phi_i(t) = \begin{cases} \sqrt{\frac{2}{T_b}} \cos(2\pi f_i t), & 0 \leq t \leq T_b \\ 0, & \text{fora} \end{cases}$$

$i \in \{1,2\}$ ,  $f_i = \frac{n_c + i}{T_b}$ ,  $n_c$ : inteiro. Notar que:  $f_2 - f_1 = \frac{1}{T_b}$

Geração e Detecção Coerente de Sinais FSK Binários

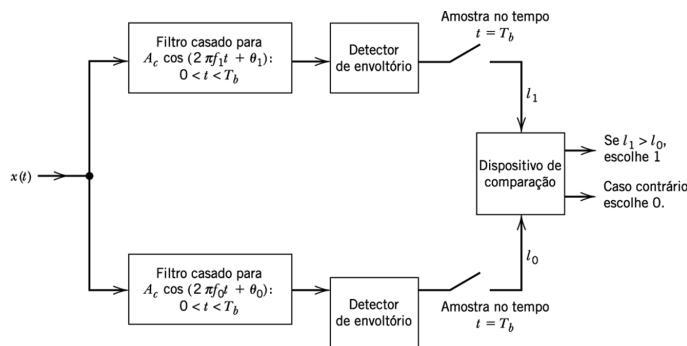


$$\text{BER} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{2N_0}}$$

## Notes

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## Receptor FSK Binário não Coerente



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$$\text{BER} = \frac{1}{2} e^{-\frac{E_b}{2N_0}}$$

## Exercícios

- ① Determine o número esperado de bits detectados erradamente ao longo de um dia por um receptor BPSK coerente funcionando continuamente. A taxa de transmissão é de 5 kbits/s. As formas de onda de entrada são  $s_1(t) = A \cos(2\pi f_{ct}t) = -s_2(t)$ , onde  $A = 1$  mV e a densidade espectral de potência de ruído é  $\frac{N_0}{2} = 10^{-11}$  W/Hz.
- ② Um sinal binário com taxa  $R_b = 2,5$  Mbits/s é aplicado na entrada de um modulador FSK. O sinal modulado é enviado por um canal AWGN com  $N_0 = 10^{-6}$  W/Hz. Na ausência de ruído e independentemente do bit enviado ser '0' ou '1' a amplitude da onda senoidal recebida é de  $1 \times 10^{-6}$  V. Calcule a taxa de erro de bits se utilizar um receptor FSK coerente.

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## Modulação FSK de Fase Contínua (CPFSK)

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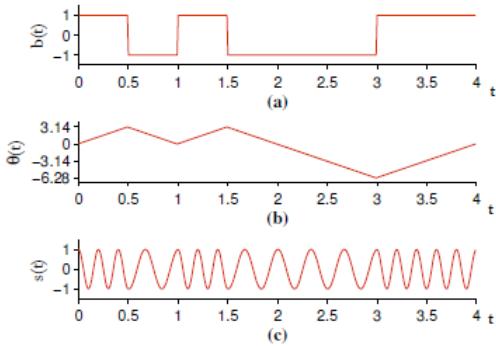
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$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos[2\pi f_c t + \theta(t)], \quad \theta(t) = \theta(0) + \frac{\pi h}{T_b} \int_0^t b(t) dt$$

$h = 1 \rightarrow \text{CPFSK}, \quad h = 0.5 \rightarrow \text{MSK}$



## Minimum Shift Keying (MSK)

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$$\begin{aligned} s(t) &= \sqrt{\frac{2E_b}{T_b}} \cos \left[ \theta(0) \pm \frac{\pi}{2T_b} t \right] \cos(2\pi f_c t) - \sqrt{\frac{2E_b}{T_b}} \sin \left[ \theta(0) \pm \frac{\pi}{2T_b} t \right] \sin(2\pi f_c t) \\ &= s_I(t) \cos(2\pi f_c t) - s_Q(t) \sin(2\pi f_c t), \quad 0 \leq t \leq 2T_b, \end{aligned}$$

onde,

$$\begin{aligned} s_I(t) &= \pm \sqrt{\frac{2E_b}{T_b}} \cos \left[ \frac{\pi}{2T_b} t \right], \quad -T_b \leq t \leq T_b, \\ s_Q(t) &= \pm \sqrt{\frac{2E_b}{T_b}} \sin \left[ \frac{\pi}{2T_b} t \right], \quad 0 \leq t \leq 2T_b. \end{aligned}$$

Considerando  $\theta(0) = 0$ ,

$$s(t) = \sqrt{\frac{2}{T_b}} a_I(t) \cos \left[ \frac{\pi}{2T_b} t \right] \cos(2\pi f_c t) - \sqrt{\frac{2}{T_b}} a_Q(t) \sin \left[ \frac{\pi}{2T_b} t \right] \sin(2\pi f_c t).$$

## Minimum Shift Keying (MSK)

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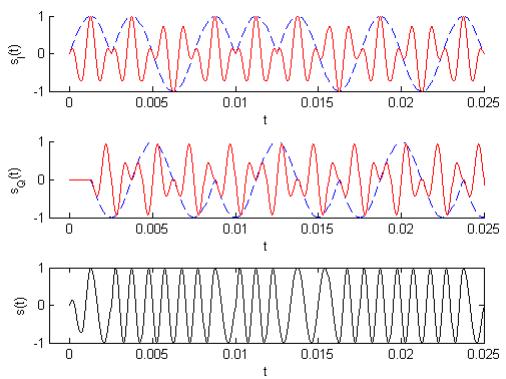
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## Espectro de Frequências – MSK

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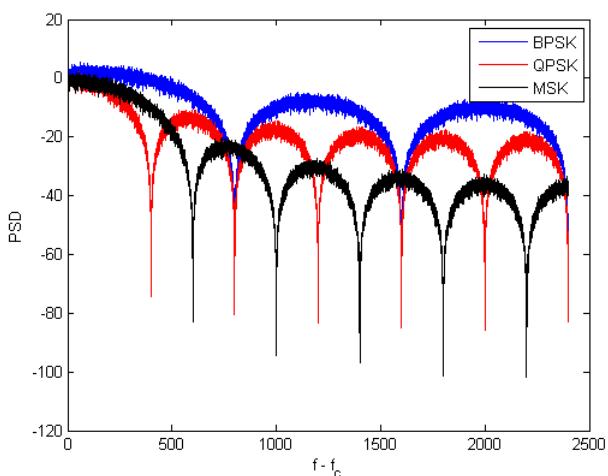
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## Gaussian Minimum Shift Keying (GMSK)

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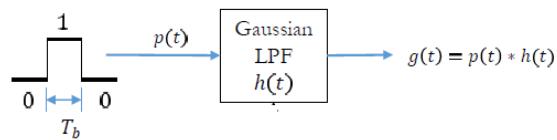
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$$h(t) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-t^2}{2\sigma^2}\right), \quad \sigma^2 = \frac{\ln(2)}{(2\pi W)^2},$$

onde  $W \rightarrow$  largura de banda de 3 dB do filtro Gaussiano.

$$\Rightarrow g(t) = \frac{1}{2T_b} \left[ Q\left(\frac{2\pi WT_b}{\sqrt{\ln 2}} \left(\frac{t}{T_b} - 1\right)\right) - Q\left(\frac{2\pi WT_b}{\sqrt{\ln 2}} \frac{t}{T_b}\right) \right],$$

$$\text{onde } Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right) dy.$$

## Gaussian Minimum Shift Keying (GMSK)

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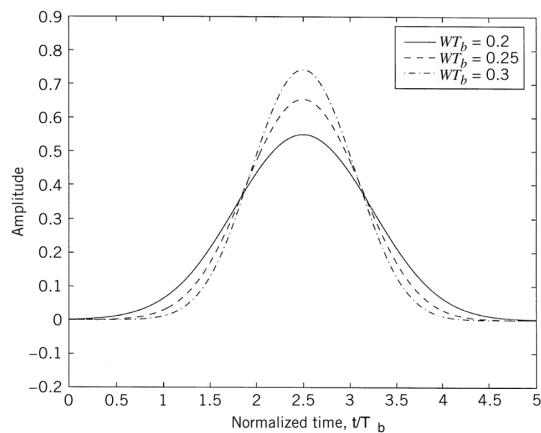
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## Espectro de Potência MSK e GMSK

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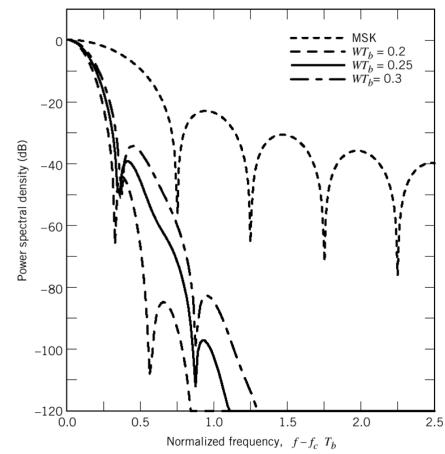
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## Espectro de Potência GSM

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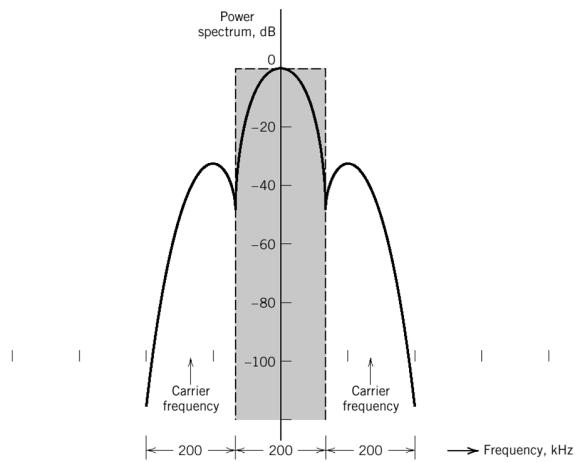
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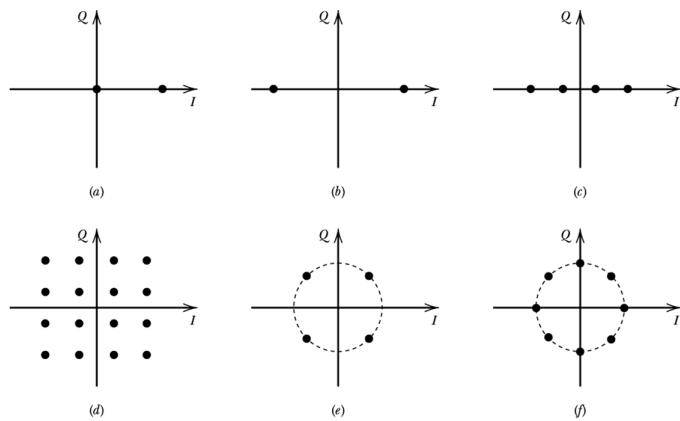
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## Sistemas de Modulação $M$ -ários



Representação de diferentes modulações passa-faixa no espaço de sinal: (a) ASK binário; (b) PSK binário; (c)  $M$ -ASK ( $M = 4$ ); (d)  $M$ -QAM ( $M = 16$ ); (e) 4-QAM e QPSK; e (f)  $M$ -PSK ( $M = 8$ ).

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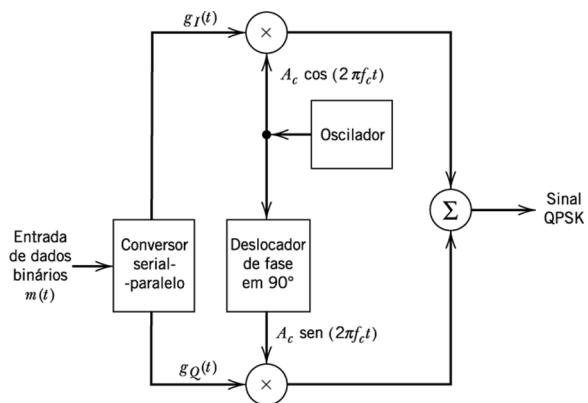
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## Chaveamento de Quadrifase (QPSK)



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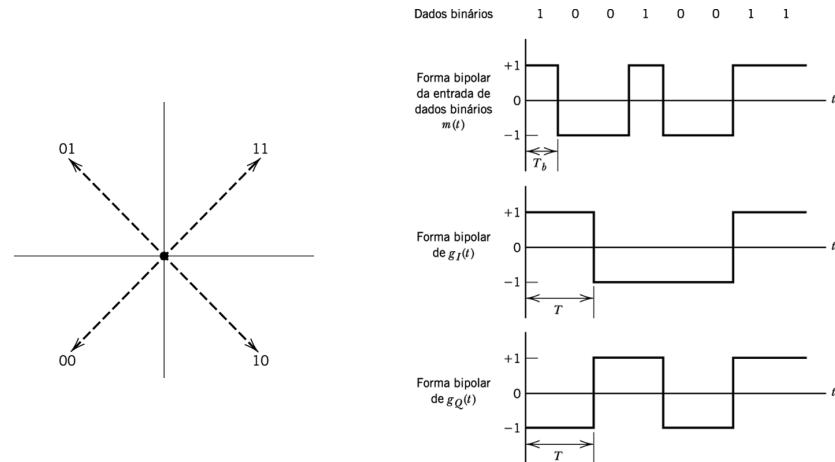
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## Chaveamento de Quadrifase (QPSK)

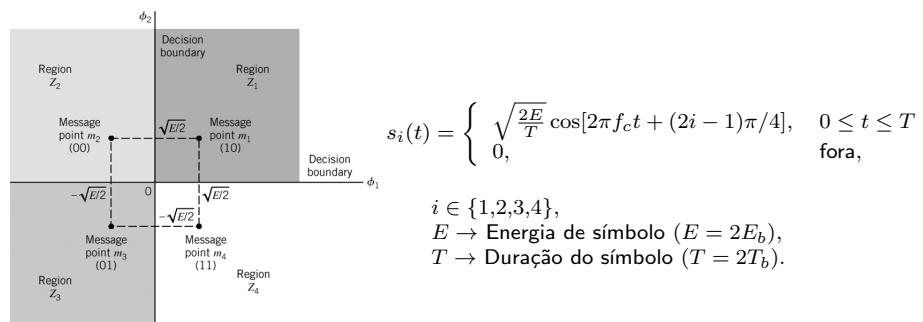
## Notes



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Espaço de Sinais – QPSK Coerente

## Notes



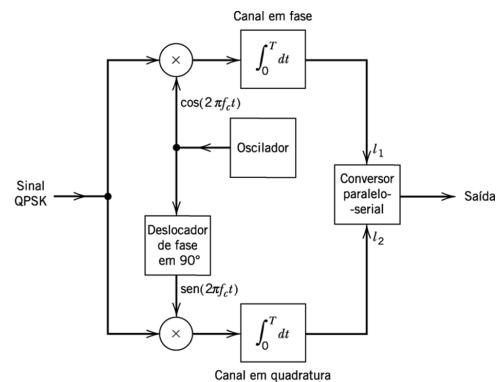
$$s_i(t) = \sqrt{\frac{2E}{T}} \cos[(2i-1)\pi/4] \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin[(2i-1)\pi/4] \sin(2\pi f_c t)$$

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t), \quad 0 \leq t \leq T$$

$$\phi_2(t) = \sqrt{\frac{2}{\pi}} \sin(2\pi f_c t), \quad 0 \leq t \leq T$$

$$\mathbf{s}_i = \begin{bmatrix} \sqrt{E} \cos[(2i-1)\pi/4] \\ -\sqrt{E} \sin[(2i-1)\pi/4] \end{bmatrix}, i = 1, 2, 3, 4.$$

## Receptor QPSK



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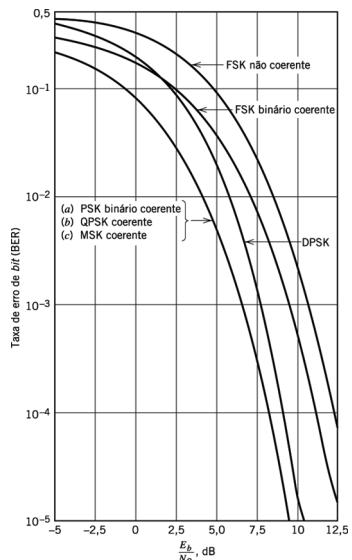
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$$\text{BER} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}}$$

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## Comparação de Desempenho de Sistemas PSK e FSK



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## QPSK Deslocado (Off-Set QPSK, OQPSK)

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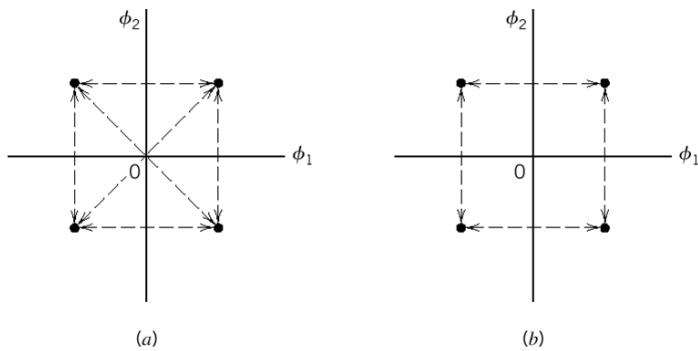
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(a)

(b)

## Modulador OQPSK

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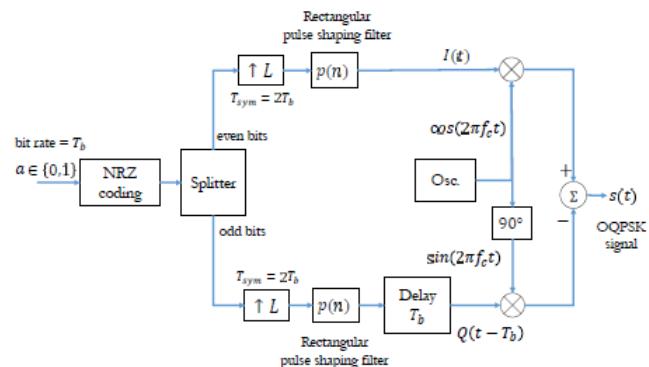
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## Demodulador OQPSK

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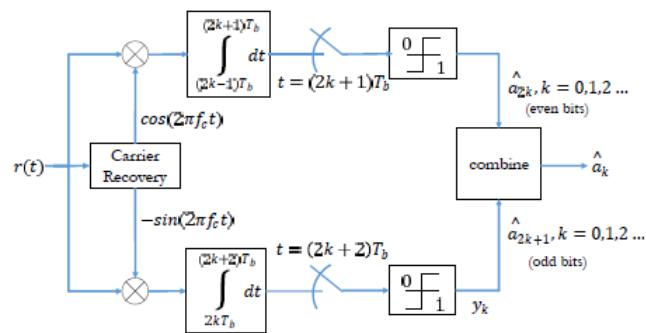
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## Duas Constelações QPSK

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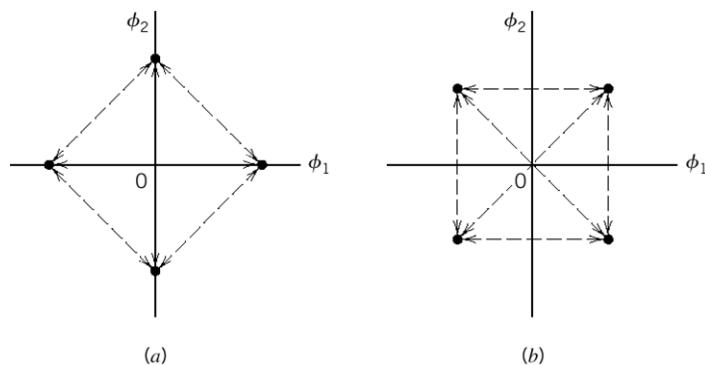
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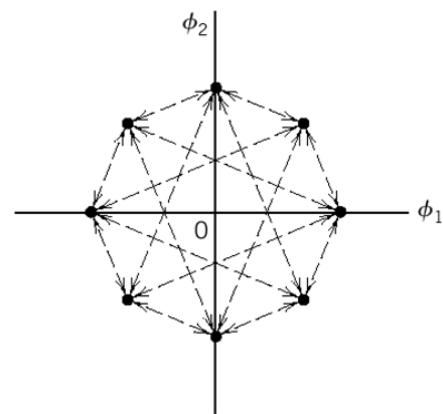
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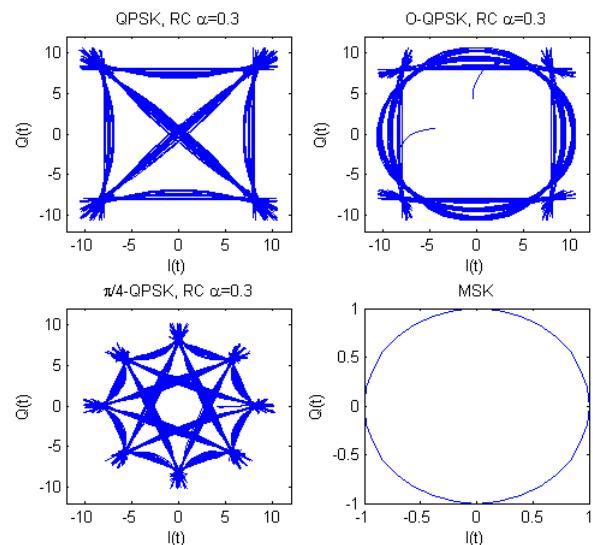
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## Constelações com Formatação Cosseno Elevado



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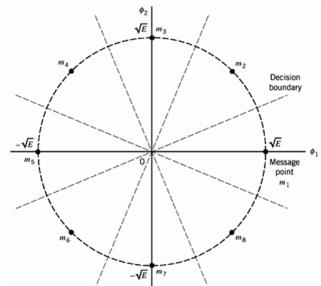
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## Espaço de Sinais M-PSK

## Constelação de Sinais 8-PSK



$$s_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{2\pi}{M}(2i-1)\right), & 0 \leq t \leq T \\ 0, & \text{fora,} \end{cases}$$

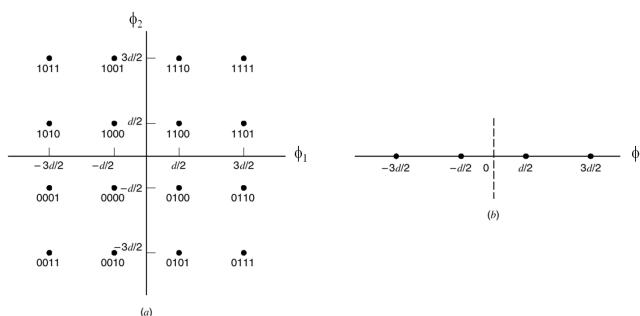
$$i = 1, 2, \dots, M, \quad M = 2^k, \\ E \rightarrow \text{Energia de símbolo } (E = kE_b), \\ T \rightarrow \text{Duração do símbolo } (T = kT_b).$$

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t), \quad 0 \leq t \leq T \quad \phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t), \quad 0 \leq t \leq T$$

$$\text{BER} \approx \frac{1}{\log_2 M} \text{erfc} \left( \sqrt{\frac{E}{N_0}} \sin \frac{\pi}{M} \right), \quad M \geq 4$$

## Notes

Espaço de Sinais M-QAM



$$s_i(t) = \sqrt{\frac{2E_0}{T}} a_i \cos(2\pi f_{ct} t) + \sqrt{\frac{2E_0}{T}} b_i \sin(2\pi f_{ct} t), \quad 0 \leq t \leq T$$

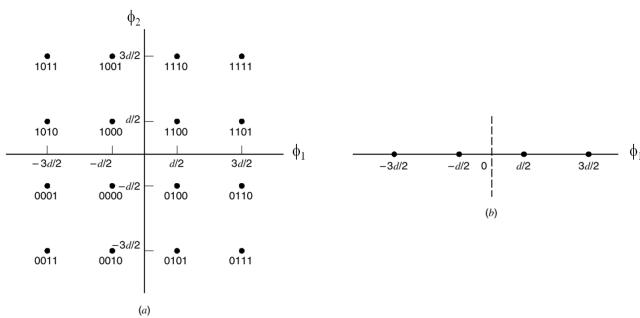
**Coordenadas dos pontos:**  $(a_i \cdot d/2, a_i \cdot d/2)$ , onde:

$$\{a_i, b_i\} \in \left\{ -\sqrt{M} + 1, -\sqrt{M} + 3, \dots, -3, -1, 1, 3, \dots, \sqrt{M} - 3, \sqrt{M} - 1 \right\} \text{ e } d/2 = \sqrt{E_0}.$$

$$\Rightarrow s_i(t) \equiv \sqrt{E_0}a_i\phi_1(t) + \sqrt{E_0}b_i\phi_2(t), \quad 0 \leq t \leq T$$

## Notes

# Desempenho de Erro de Modulações M-QAM



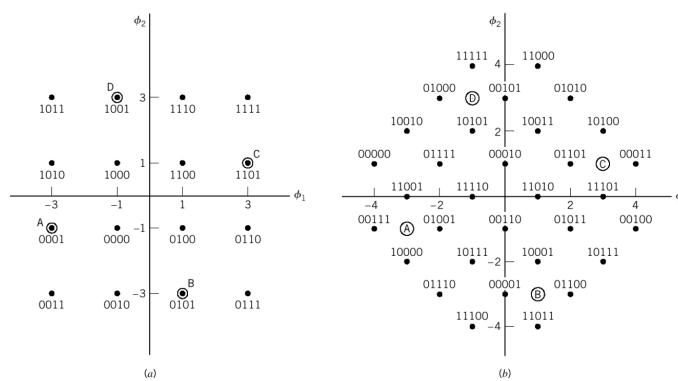
$$\begin{aligned}\texttt{BER} &\approx \frac{1}{\log_2 M} 2 \left( 1 - \frac{1}{\sqrt{M}} \right) \operatorname{erfc} \left( \sqrt{\frac{3E_s}{2(M-1)N_0}} \right) \\ &\approx \frac{1}{\log_2 M} 2 \left( 1 - \frac{1}{\sqrt{M}} \right) \operatorname{erfc} \left( \sqrt{\frac{E_0}{N_0}} \right)\end{aligned}$$

onde  $\overline{E_s} = \frac{2(M-1)E_0}{3}$ ,  $E_0 = \left(\frac{d}{2}\right)^2$

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## Notes



## Notes

## Constelação V.34

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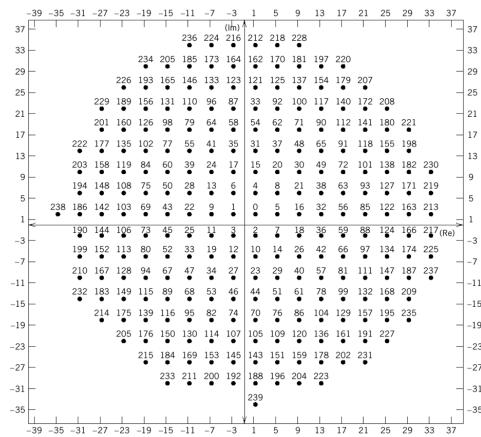
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## Modulação M-FSK

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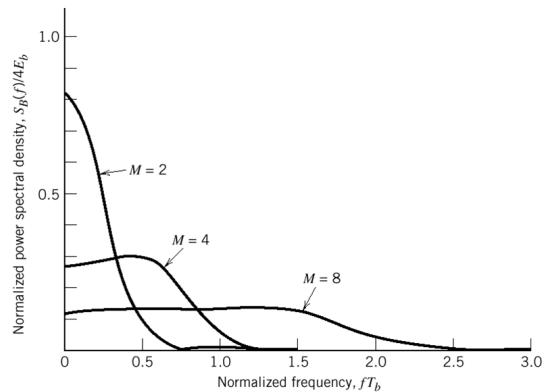
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$$P_e \leq \frac{1}{2}(M-1)\operatorname{erfc}\left(\sqrt{\frac{E_s}{2N_0}}\right), \quad \text{BER} = \left(\frac{M/2}{M-1}\right) P_e$$

Evelio M. G. Fernández

TE903 – Comunicação Digital

## Desempenho de Erro de Sistemas de Modulação Digital

Notes

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Modulação	Taxa de Erro de Bits
PSK binário coerente	$BER = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}}$
DPSK	$BER = \frac{1}{2} e^{-\frac{E_b}{N_0}}$
FSK binário coerente	$BER = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{2N_0}}$
FSK binário não coerente	$BER = \frac{1}{2} e^{-\frac{E_b}{2N_0}}$
QPSK	$BER = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}}$
M-PSK	$BER \approx \frac{1}{\log_2 M} \operatorname{erfc} \left( \sqrt{\frac{E_s}{N_0}} \sin \frac{\pi}{M} \right)$
M-QAM	$BER \approx \frac{1}{\log_2 M} 2 \left( 1 - \frac{1}{\sqrt{M}} \right) \operatorname{erfc} \left( \sqrt{\frac{3E_s}{2(M-1)N_0}} \right)$

Evelio M. G. Fernández

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## Desempenho de Erro de Modulações Digitais

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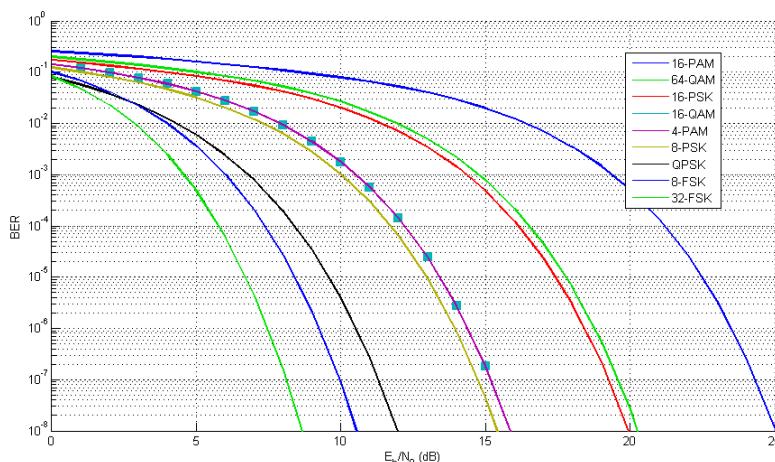
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## Exercício

Considere um sistema de comunicação em banda passante que transmite símbolos  $M$ -QAM a uma taxa de  $R_s = 5\text{M}$  símbolos/seg sobre um canal Gaussiano com  $\overline{E_s}/N_0 = 27,5 \text{ dB}$ . Deseja-se que a probabilidade média de erro de símbolos deste sistema seja  $P_e \leq 10^{-6}$ .

- a) Qual será a máxima taxa de transmissão em bits/seg que pode ser atingida?
- b) Se quisermos aumentar a taxa de transmissão em 5 Mbps, qual terá que ser a relação sinal-ruído ( $\overline{E_s}/N_0$ ) sem alterar os valores de taxa de símbolos e probabilidade de erro de símbolos?

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## Exercício

Queremos projetar um sistema que seja capaz de transmitir  $R_b = 7500 \text{ bits/seg}$  através de um canal AWGN com  $N_0 = 10^{-8} \text{ W/Hz}$ . A taxa de erro de bits deve ser  $\text{BER} \leq 10^{-5}$ . Deseja-se que o sistema seja o mais eficiente possível tanto em potência quanto espectralmente.

- a) Determine o diagrama de espaço de sinal a ser utilizado considerando que o canal de comunicação é passa-baixas com 1kHz de largura de banda.
- b) Repita o item (a) supondo um canal passa-faixa com 1kHz de largura de banda.
- c) Compare a eficiência de potência e a eficiência espectral em (a) e (b). Comente.

Notes

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