EELT-7035 – Processos Estocásticos em Engenharia

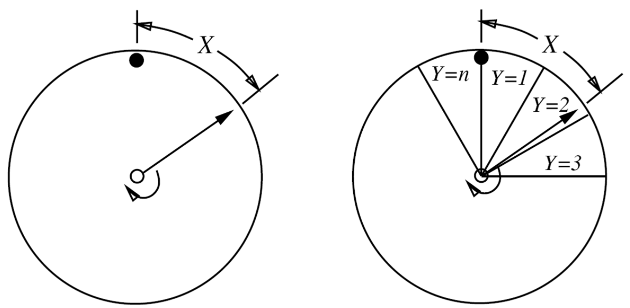
Variáveis Aleatórias Contínuas

Evelio M. G. Fernández

20 de março de 2020

Notes

Ponteiro Aleatório num Disco de Circunferência $= 1$



Notes

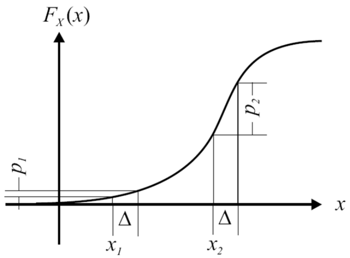
Função de Distribuição Acumulada, CDF

Notes

- **Função de Distribuição Acumulada:** $F_X(x) = P[X \leq x]$
- Para qualquer variável aleatória X,
  - (a) $F_X(-\infty) = 0$
  - (b) $F_X(\infty) = 1$
  - (c) $\mathcal{P}[x_1 < X \leq x_2] = F_X(x_2) - F_X(x_1)$.
- X é uma variável aleatória contínua se a CDF $F_X(x)$ é uma função contínua.

Função Densidade de Probabilidade, PDF

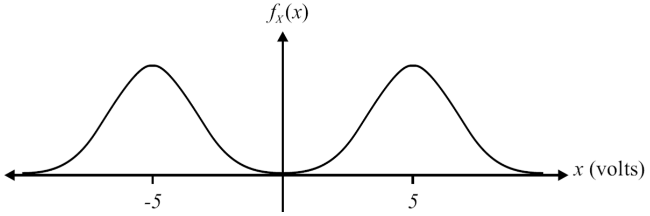
Notes



- $p_1 = \mathcal{P}[x_1 < X \leq x_1 + \Delta] = F_X(x_1 + \Delta) - F_X(x_1)$
- $p_2 = \mathcal{P}[x_2 < X \leq x_2 + \Delta] = F_X(x_2 + \Delta) - F_X(x_2)$
- $\mathcal{P}[x_1 < X \leq x_1 + \Delta] = \dfrac{F_X(x_1 + \Delta) - F_X(x_1)}{\Delta}\Delta \Rightarrow$ a probabilidade de X estar num intervalo perto de x_1 é igual à inclinação média de $F_X(x)$ vezes os tamanho do intervalo.
- **Função Densidade de Probabilidade:**

$$f_X(x) = \frac{dF_X(x)}{dx}$$

Propriedades da Função Densidade de Probabilidade

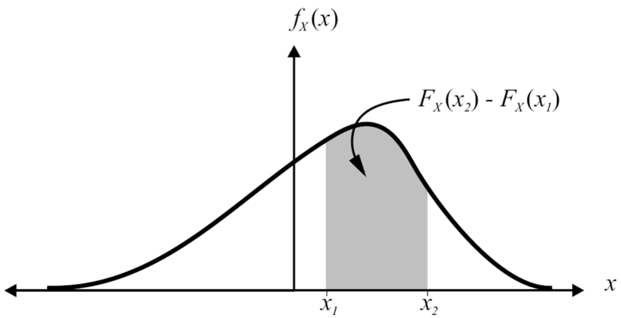


Para uma variável aleatória X com PDF $f_X(x)$,

- (a) $f_X(x) \geq 0$ para todo x,
- (b) $F_X(x) = \int_{-\infty}^{x} f_X(u)du$,
- (c) $\int_{-\infty}^{\infty} f_X(x)dx = 1$
- (d) $\mathcal{P}[x_1 < X \leq x_2] = \int_{x_1}^{x_2} f_X(x)dx$

Notes

PDF e CDF



$$\mathcal{P}[x_1 < X \leq x_2] = \mathcal{P}[X \leq x_2] - \mathcal{P}[X \leq x_1] = F_X(x_2) - F_X(x_1) = \int_{x_1}^{x_2} f_X(x)dx$$

Lembrando que: $\mathcal{P}[x_1 < X \leq x_1 + \Delta] = \dfrac{F_X(x_1 + \Delta) - F_X(x_1)}{\Delta}\Delta$

$\Rightarrow \mathcal{P}[x < X \leq x + dx] = f_X(x)dx$

Notes

Exercício 1

Considere a variável aleatória X com função densidade de probabilidade dada por:

$$f_X(x) = \begin{cases} cxe^{-x/2}, & x \geq 0, \\ 0, & \text{fora}. \end{cases}$$

Determine:

a) O valor de c
b) $F_X(x)$
c) $\mathcal{P}[0 \leq X \leq 4]$
d) $\mathcal{P}[-2 \leq X \leq 2]$

Notes

Momentos

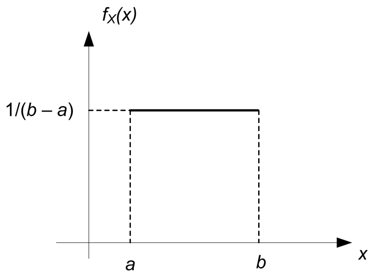
- **Valor Esperado:** $E[X] = \int_{-\infty}^{\infty} x f_X(x) dx$
- **Valor Esperado de $g(X)$:** $E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x) dx$
- **Valor Médio Quadrático:** $E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x) dx$
- **Variância:** $\mathrm{Var}[X] = \int_{-\infty}^{\infty} (x - \mu_x)^2 f_X(x) dx$

Notes

Exercício 2

Determine a média e a variância da variável aleatória contínua X com distribuição uniforme dada por:

$$f_X(x) = \begin{cases} \dfrac{1}{b-a}, & a \leq x \leq b, \\ 0, & \text{fora.} \end{cases}$$



Notes

Variável Aleatória Uniforme (a,b)

- $f_X(x) = \begin{cases} 1/(b-a), & a \leq x < b, \\ 0, & \text{fora,} \end{cases} \quad b > a.$
  - $F_X(x) = \begin{cases} 0, & x \leq a, \\ (x-a)/(b-a), & a \leq x < b, \\ 1, & x > b. \end{cases}$
  - $E[X] = (b+a)/2$
  - $\text{Var}[X] = (b-a)^2/12$
  - Seja X uma V.A. uniforme (a,b), onde a e b são inteiros. Então, a V.A. $K = \lceil X \rceil$ é discreta uniforme $(a+1,b)$

Notes

Exercício 3

A variável aleatória X tem PDF dada por:

$$f_X(x) = \begin{cases} 1/8, & 1 \leq x \leq 9, \\ 0, & \text{fora.} \end{cases}$$

Seja $Y = h(X) = \frac{1}{\sqrt{X}}$.

Determine:

a) $E[X]$ e $\mathrm{Var}[X]$;
b) $h(E[X])$ e $E[h(X)]$;
c) $E[Y]$ e $\mathrm{Var}[Y]$.

Notes

Variável Aleatória Exponencial (λ)

- $f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \\ 0, & \text{fora,} \end{cases} \quad \lambda > 0.$
  - $F_X(x) = \begin{cases} 1 - e^{-\lambda x}, & x \geq 0, \\ 0, & \text{fora.} \end{cases}$
  - $E[X] = 1/\lambda$
  - $\mathrm{Var}[X] = 1/\lambda^2$
  - Seja X uma V.A. exponencial (λ). Então, a V.A. $K = \lceil X \rceil$ é geométrica (p) com $p = 1 - e^{-\lambda}$

Notes

Exemplo

- **Operadora A** cobra R$ 0,15/minuto nas chamadas telefônicas. A fração de minuto ao finalizar a chamada é cobrada como sendo um minuto;
- **Operadora B** também cobra R$ 0,15/minuto. Porém, o valor da chamada é baseado na duração exta da chamada;
- Considere a V.A. $T \to$ duração de uma chamada minutos, como sendo exponencial $(\lambda = 1/3)$;
- Valor esperado do faturamento por chamada das operadoras **A** e **B**?

Notes

Variável Aleatória de Erlang (n, λ)

- $f_X(x) = \begin{cases} \dfrac{\lambda^n x^{n-1} e^{-\lambda x}}{(n-1)!}, & x \geq 0, \\ 0, & \text{fora,} \end{cases} \qquad \lambda > 0$ e $n \geq 1$.
  - $E[X] = n/\lambda$
  - $\mathrm{Var}[X] = n/\lambda^2$
  - Seja K_α uma V.A. de Poisson (α). Então, para qualquer $x > 0$ a CDF de uma V.A. X de Erlang (n, λ) satisfaz:

$$F_X(x) = 1 - F_{K_{\lambda x}}(n-1) = 1 - \sum_{k=0}^{n-1} \frac{(\lambda x)^k e^{-\lambda x}}{k!}$$

Notes

Variável Aleatória com Distribuição Gama (α, λ)

- $$f_X(x) = \begin{cases} \dfrac{\lambda e^{-\lambda x}(\lambda x)^{\alpha-1}}{\Gamma(\alpha)}, & x \geq 0, \\ 0, & x < 0, \end{cases} \qquad \lambda > 0, \alpha > 0.$$
  - λ: parâmetro de forma, α: parâmetro de escala.
  - $\Gamma(\alpha) = \int_0^\infty e^{-y} y^{\alpha-1} dy$
  - $E[X] = \alpha/\lambda$
  - $\mathrm{Var}[X] = \alpha/\lambda^2$

Notes

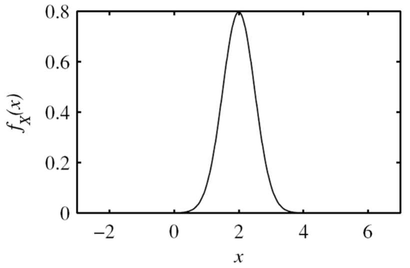
Variável Aleatória com Distribuição de Weibull (ν, α, β)

- $$f_X(x) = \begin{cases} 0, & x \leq \nu, \\ \dfrac{\beta}{\alpha}\left(\dfrac{x-\nu}{\alpha}\right)^{\beta-1} \exp\left\{-\left(\dfrac{x-\nu}{\alpha}\right)^{\beta}\right\}, & x > \nu. \end{cases}$$
  - Parâmetros: ν $(-\infty < \nu < \infty)$, de locação, $\alpha > 0$ de escala e $\beta > 0$ de forma.
  - $$F_X(x) = \begin{cases} 0, & x \leq \nu, \\ 1 - \exp\left\{-\left(\dfrac{x-\nu}{\alpha}\right)^{\beta}\right\}, & x > \nu. \end{cases}$$
  - $E[X] = \nu + \alpha\Gamma\left(1 + \frac{1}{\beta}\right)$
  - $\mathrm{Var}[X] = \alpha^2\left\{\Gamma\left(1 + \frac{2}{\beta}\right) - \left[\Gamma\left(1 + \frac{1}{\beta}\right)\right]^2\right\}$

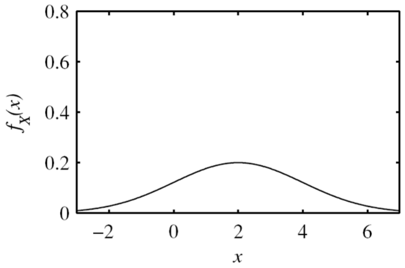
Notes

Variável Aleatória Gaussiana (μ, σ)

- $f_X(x) = \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}, \quad \mu \in \mathbb{R},\ \sigma > 0$
- $E[X] = \mu \quad \mathrm{Var}[X] = \sigma^2$



(a) $\mu = 2, \sigma = 1/2$



(b) $\mu = 2, \sigma = 2$

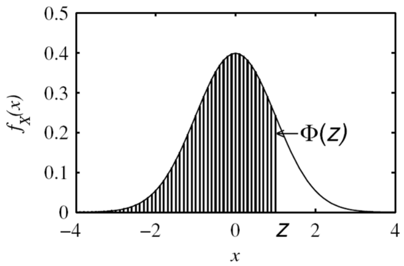
Notes

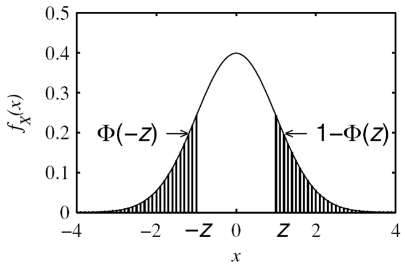
Variável Aleatória Gaussiana (μ, σ) – Propriedades

- Se X é Gaussiana (μ, σ), então $Y = aX + b$ é Gaussiana $(a\mu + b, a\sigma)$
- A V.A. normal padrão Z é a V.A. Gaussiana $(0, 1)$
- A CDF da V.A. normal padrão é: $\Phi(z) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\infty}^{z} e^{-u^2/2} du$
- A CDF de X, Gaussiana (μ, σ) é: $F_X(x) = \Phi\left(\dfrac{x-\mu}{\sigma}\right)$
- $\mathcal{P}[a < X \le b] = \Phi\left(\dfrac{b-\mu}{\sigma}\right) - \Phi\left(\dfrac{a-\mu}{\sigma}\right)$

Notes

Simetria de $\mathcal{N}(0,1)$





- $\Phi(-z) = 1 - \Phi(z) = Q(z)$
- $Q(z) = P[Z > z] = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_z^{\infty} e^{-u^2/2} du = 1 - \Phi(z)$

Notes

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
0.00	0.5000	0.50	0.6915	1.00	0.8413	1.50	0.9332	2.00	0.97725	2.50	0.99379
0.01	0.5040	0.51	0.6950	1.01	0.8438	1.51	0.9345	2.01	0.97778	2.51	0.99396
0.02	0.5080	0.52	0.6985	1.02	0.8461	1.52	0.9357	2.02	0.97831	2.52	0.99413
0.03	0.5120	0.53	0.7019	1.03	0.8485	1.53	0.9370	2.03	0.97882	2.53	0.99430
0.04	0.5160	0.54	0.7054	1.04	0.8508	1.54	0.9382	2.04	0.97932	2.54	0.99446
0.05	0.5199	0.55	0.7088	1.05	0.8531	1.55	0.9394	2.05	0.97982	2.55	0.99461
0.06	0.5239	0.56	0.7123	1.06	0.8554	1.56	0.9406	2.06	0.98030	2.56	0.99477
0.07	0.5279	0.57	0.7157	1.07	0.8577	1.57	0.9418	2.07	0.98077	2.57	0.99492
0.08	0.5319	0.58	0.7190	1.08	0.8599	1.58	0.9429	2.08	0.98124	2.58	0.99506
0.09	0.5359	0.59	0.7224	1.09	0.8621	1.59	0.9441	2.09	0.98169	2.59	0.99520
0.10	0.5398	0.60	0.7257	1.10	0.8643	1.60	0.9452	2.10	0.98214	2.60	0.99534
0.11	0.5438	0.61	0.7291	1.11	0.8665	1.61	0.9463	2.11	0.98257	2.61	0.99547
0.12	0.5478	0.62	0.7324	1.12	0.8686	1.62	0.9474	2.12	0.98300	2.62	0.99560
0.13	0.5517	0.63	0.7357	1.13	0.8708	1.63	0.9484	2.13	0.98341	2.63	0.99573
0.14	0.5557	0.64	0.7389	1.14	0.8729	1.64	0.9495	2.14	0.98382	2.64	0.99585
0.15	0.5596	0.65	0.7422	1.15	0.8749	1.65	0.9505	2.15	0.98422	2.65	0.99598
0.16	0.5636	0.66	0.7454	1.16	0.8770	1.66	0.9515	2.16	0.98461	2.66	0.99609
0.17	0.5675	0.67	0.7486	1.17	0.8790	1.67	0.9525	2.17	0.98500	2.67	0.99621
0.18	0.5714	0.68	0.7517	1.18	0.8810	1.68	0.9535	2.18	0.98537	2.68	0.99632
0.19	0.5753	0.69	0.7549	1.19	0.8830	1.69	0.9545	2.19	0.98574	2.69	0.99643
0.20	0.5793	0.70	0.7580	1.20	0.8849	1.70	0.9554	2.20	0.98610	2.70	0.99653
0.21	0.5832	0.71	0.7611	1.21	0.8869	1.71	0.9564	2.21	0.98645	2.71	0.99664
0.22	0.5871	0.72	0.7642	1.22	0.8888	1.72	0.9573	2.22	0.98679	2.72	0.99674
0.23	0.5910	0.73	0.7673	1.23	0.8907	1.73	0.9582	2.23	0.98713	2.73	0.99683
0.24	0.5948	0.74	0.7704	1.24	0.8925	1.74	0.9591	2.24	0.98745	2.74	0.99693
0.25	0.5987	0.75	0.7734	1.25	0.8944	1.75	0.9599	2.25	0.98778	2.75	0.99702
0.26	0.6026	0.76	0.7764	1.26	0.8962	1.76	0.9608	2.26	0.98809	2.76	0.99711
0.27	0.6064	0.77	0.7794	1.27	0.8980	1.77	0.9616	2.27	0.98840	2.77	0.99720
0.28	0.6103	0.78	0.7823	1.28	0.8997	1.78	0.9625	2.28	0.98870	2.78	0.99728
0.29	0.6141	0.79	0.7852	1.29	0.9015	1.79	0.9633	2.29	0.98899	2.79	0.99736
0.30	0.6179	0.80	0.7881	1.30	0.9032	1.80	0.9641	2.30	0.98928	2.80	0.99744
0.31	0.6217	0.81	0.7910	1.31	0.9049	1.81	0.9649	2.31	0.98956	2.81	0.99752
0.32	0.6255	0.82	0.7939	1.32	0.9066	1.82	0.9656	2.32	0.98983	2.82	0.99760
0.33	0.6293	0.83	0.7967	1.33	0.9082	1.83	0.9664	2.33	0.99010	2.83	0.99767
0.34	0.6331	0.84	0.7995	1.34	0.9099	1.84	0.9671	2.34	0.99036	2.84	0.99774
0.35	0.6368	0.85	0.8023	1.35	0.9115	1.85	0.9678	2.35	0.99061	2.85	0.99781
0.36	0.6406	0.86	0.8051	1.36	0.9131	1.86	0.9686	2.36	0.99086	2.86	0.99788
0.37	0.6443	0.87	0.8078	1.37	0.9147	1.87	0.9693	2.37	0.99111	2.87	0.99795
0.38	0.6480	0.88	0.8106	1.38	0.9162	1.88	0.9699	2.38	0.99134	2.88	0.99801
0.39	0.6517	0.89	0.8133	1.39	0.9177	1.89	0.9706	2.39	0.99158	2.89	0.99807
0.40	0.6554	0.90	0.8159	1.40	0.9192	1.90	0.9713	2.40	0.99180	2.90	0.99813
0.41	0.6591	0.91	0.8186	1.41	0.9207	1.91	0.9719	2.41	0.99202	2.91	0.99819
0.42	0.6628	0.92	0.8212	1.42	0.9222	1.92	0.9726	2.42	0.99224	2.92	0.99825
0.43	0.6664	0.93	0.8238	1.43	0.9236	1.93	0.9732	2.43	0.99245	2.93	0.99831
0.44	0.6700	0.94	0.8264	1.44	0.9251	1.94	0.9738	2.44	0.99266	2.94	0.99836
0.45	0.6736	0.95	0.8289	1.45	0.9265	1.95	0.9744	2.45	0.99286	2.95	0.99841
0.46	0.6772	0.96	0.8315	1.46	0.9279	1.96	0.9750	2.46	0.99305	2.96	0.99846
0.47	0.6808	0.97	0.8340	1.47	0.9292	1.97	0.9756	2.47	0.99324	2.97	0.99851
0.48	0.6844	0.98	0.8365	1.48	0.9306	1.98	0.9761	2.48	0.99343	2.98	0.99856
0.49	0.6879	0.99	0.8389	1.49	0.9319	1.99	0.9767	2.49	0.99361	2.99	0.99861

Notes

z	$Q(z)$	z	$Q(z)$	z	$Q(z)$	z	$Q(z)$	z	$Q(z)$
3.00	$1.35\cdot10^{-3}$	3.40	$3.37\cdot10^{-4}$	3.80	$7.23\cdot10^{-5}$	4.20	$1.33\cdot10^{-5}$	4.60	$2.11\cdot10^{-6}$
3.01	$1.31\cdot10^{-3}$	3.41	$3.25\cdot10^{-4}$	3.81	$6.95\cdot10^{-5}$	4.21	$1.28\cdot10^{-5}$	4.61	$2.01\cdot10^{-6}$
3.02	$1.26\cdot10^{-3}$	3.42	$3.13\cdot10^{-4}$	3.82	$6.67\cdot10^{-5}$	4.22	$1.22\cdot10^{-5}$	4.62	$1.92\cdot10^{-6}$
3.03	$1.22\cdot10^{-3}$	3.43	$3.02\cdot10^{-4}$	3.83	$6.41\cdot10^{-5}$	4.23	$1.17\cdot10^{-5}$	4.63	$1.83\cdot10^{-6}$
3.04	$1.18\cdot10^{-3}$	3.44	$2.91\cdot10^{-4}$	3.84	$6.15\cdot10^{-5}$	4.24	$1.12\cdot10^{-5}$	4.64	$1.74\cdot10^{-6}$
3.05	$1.14\cdot10^{-3}$	3.45	$2.80\cdot10^{-4}$	3.85	$5.91\cdot10^{-5}$	4.25	$1.07\cdot10^{-5}$	4.65	$1.66\cdot10^{-6}$
3.06	$1.11\cdot10^{-3}$	3.46	$2.70\cdot10^{-4}$	3.86	$5.67\cdot10^{-5}$	4.26	$1.02\cdot10^{-5}$	4.66	$1.58\cdot10^{-6}$
3.07	$1.07\cdot10^{-3}$	3.47	$2.60\cdot10^{-4}$	3.87	$5.44\cdot10^{-5}$	4.27	$9.77\cdot10^{-6}$	4.67	$1.51\cdot10^{-6}$
3.08	$1.04\cdot10^{-3}$	3.48	$2.51\cdot10^{-4}$	3.88	$5.22\cdot10^{-5}$	4.28	$9.34\cdot10^{-6}$	4.68	$1.43\cdot10^{-6}$
3.09	$1.00\cdot10^{-3}$	3.49	$2.42\cdot10^{-4}$	3.89	$5.01\cdot10^{-5}$	4.29	$8.93\cdot10^{-6}$	4.69	$1.37\cdot10^{-6}$
3.10	$9.68\cdot10^{-4}$	3.50	$2.33\cdot10^{-4}$	3.90	$4.81\cdot10^{-5}$	4.30	$8.54\cdot10^{-6}$	4.70	$1.30\cdot10^{-6}$
3.11	$9.35\cdot10^{-4}$	3.51	$2.24\cdot10^{-4}$	3.91	$4.61\cdot10^{-5}$	4.31	$8.16\cdot10^{-6}$	4.71	$1.24\cdot10^{-6}$
3.12	$9.04\cdot10^{-4}$	3.52	$2.16\cdot10^{-4}$	3.92	$4.43\cdot10^{-5}$	4.32	$7.80\cdot10^{-6}$	4.72	$1.18\cdot10^{-6}$
3.13	$8.74\cdot10^{-4}$	3.53	$2.08\cdot10^{-4}$	3.93	$4.25\cdot10^{-5}$	4.33	$7.46\cdot10^{-6}$	4.73	$1.12\cdot10^{-6}$
3.14	$8.45\cdot10^{-4}$	3.54	$2.00\cdot10^{-4}$	3.94	$4.07\cdot10^{-5}$	4.34	$7.12\cdot10^{-6}$	4.74	$1.07\cdot10^{-6}$
3.15	$8.16\cdot10^{-4}$	3.55	$1.93\cdot10^{-4}$	3.95	$3.91\cdot10^{-5}$	4.35	$6.81\cdot10^{-6}$	4.75	$1.02\cdot10^{-6}$
3.16	$7.89\cdot10^{-4}$	3.56	$1.85\cdot10^{-4}$	3.96	$3.75\cdot10^{-5}$	4.36	$6.50\cdot10^{-6}$	4.76	$9.68\cdot10^{-7}$
3.17	$7.62\cdot10^{-4}$	3.57	$1.78\cdot10^{-4}$	3.97	$3.59\cdot10^{-5}$	4.37	$6.21\cdot10^{-6}$	4.77	$9.21\cdot10^{-7}$
3.18	$7.36\cdot10^{-4}$	3.58	$1.72\cdot10^{-4}$	3.98	$3.45\cdot10^{-5}$	4.38	$5.93\cdot10^{-6}$	4.78	$8.76\cdot10^{-7}$
3.19	$7.11\cdot10^{-4}$	3.59	$1.65\cdot10^{-4}$	3.99	$3.30\cdot10^{-5}$	4.39	$5.67\cdot10^{-6}$	4.79	$8.34\cdot10^{-7}$
3.20	$6.87\cdot10^{-4}$	3.60	$1.59\cdot10^{-4}$	4.00	$3.17\cdot10^{-5}$	4.40	$5.41\cdot10^{-6}$	4.80	$7.93\cdot10^{-7}$
3.21	$6.64\cdot10^{-4}$	3.61	$1.53\cdot10^{-4}$	4.01	$3.04\cdot10^{-5}$	4.41	$5.17\cdot10^{-6}$	4.81	$7.55\cdot10^{-7}$
3.22	$6.41\cdot10^{-4}$	3.62	$1.47\cdot10^{-4}$	4.02	$2.91\cdot10^{-5}$	4.42	$4.94\cdot10^{-6}$	4.82	$7.18\cdot10^{-7}$
3.23	$6.19\cdot10^{-4}$	3.63	$1.42\cdot10^{-4}$	4.03	$2.79\cdot10^{-5}$	4.43	$4.71\cdot10^{-6}$	4.83	$6.83\cdot10^{-7}$
3.24	$5.98\cdot10^{-4}$	3.64	$1.36\cdot10^{-4}$	4.04	$2.67\cdot10^{-5}$	4.44	$4.50\cdot10^{-6}$	4.84	$6.49\cdot10^{-7}$
3.25	$5.77\cdot10^{-4}$	3.65	$1.31\cdot10^{-4}$	4.05	$2.56\cdot10^{-5}$	4.45	$4.29\cdot10^{-6}$	4.85	$6.17\cdot10^{-7}$
3.26	$5.57\cdot10^{-4}$	3.66	$1.26\cdot10^{-4}$	4.06	$2.45\cdot10^{-5}$	4.46	$4.10\cdot10^{-6}$	4.86	$5.87\cdot10^{-7}$
3.27	$5.38\cdot10^{-4}$	3.67	$1.21\cdot10^{-4}$	4.07	$2.35\cdot10^{-5}$	4.47	$3.91\cdot10^{-6}$	4.87	$5.58\cdot10^{-7}$
3.28	$5.19\cdot10^{-4}$	3.68	$1.17\cdot10^{-4}$	4.08	$2.25\cdot10^{-5}$	4.48	$3.73\cdot10^{-6}$	4.88	$5.30\cdot10^{-7}$
3.29	$5.01\cdot10^{-4}$	3.69	$1.12\cdot10^{-4}$	4.09	$2.16\cdot10^{-5}$	4.49	$3.56\cdot10^{-6}$	4.89	$5.04\cdot10^{-7}$
3.30	$4.83\cdot10^{-4}$	3.70	$1.08\cdot10^{-4}$	4.10	$2.07\cdot10^{-5}$	4.50	$3.40\cdot10^{-6}$	4.90	$4.79\cdot10^{-7}$
3.31	$4.66\cdot10^{-4}$	3.71	$1.04\cdot10^{-4}$	4.11	$1.98\cdot10^{-5}$	4.51	$3.24\cdot10^{-6}$	4.91	$4.55\cdot10^{-7}$
3.32	$4.50\cdot10^{-4}$	3.72	$9.96\cdot10^{-5}$	4.12	$1.89\cdot10^{-5}$	4.52	$3.09\cdot10^{-6}$	4.92	$4.33\cdot10^{-7}$
3.33	$4.34\cdot10^{-4}$	3.73	$9.57\cdot10^{-5}$	4.13	$1.81\cdot10^{-5}$	4.53	$2.95\cdot10^{-6}$	4.93	$4.11\cdot10^{-7}$
3.34	$4.19\cdot10^{-4}$	3.74	$9.20\cdot10^{-5}$	4.14	$1.74\cdot10^{-5}$	4.54	$2.81\cdot10^{-6}$	4.94	$3.91\cdot10^{-7}$
3.35	$4.04\cdot10^{-4}$	3.75	$8.84\cdot10^{-5}$	4.15	$1.66\cdot10^{-5}$	4.55	$2.68\cdot10^{-6}$	4.95	$3.71\cdot10^{-7}$
3.36	$3.90\cdot10^{-4}$	3.76	$8.50\cdot10^{-5}$	4.16	$1.59\cdot10^{-5}$	4.56	$2.56\cdot10^{-6}$	4.96	$3.52\cdot10^{-7}$
3.37	$3.76\cdot10^{-4}$	3.77	$8.16\cdot10^{-5}$	4.17	$1.52\cdot10^{-5}$	4.57	$2.44\cdot10^{-6}$	4.97	$3.35\cdot10^{-7}$
3.38	$3.62\cdot10^{-4}$	3.78	$7.84\cdot10^{-5}$	4.18	$1.46\cdot10^{-5}$	4.58	$2.32\cdot10^{-6}$	4.98	$3.18\cdot10^{-7}$
3.39	$3.49\cdot10^{-4}$	3.79	$7.53\cdot10^{-5}$	4.19	$1.39\cdot10^{-5}$	4.59	$2.22\cdot10^{-6}$	4.99	$3.02\cdot10^{-7}$

Notes

Exemplos V.A. Gaussianas

1. Suponha que a nota de um aluno num teste, $x = 46$, é uma amostra de V.A. Gaussiana (61,10).
   - Expressar este resultado como uma amostra de V.A. padrão normal, Z;
   - $\mathcal{P}[X \leq 46]$?
2. Seja X uma V.A. Gaussiana com $\mu = 61$ e $\sigma = 10$.
   - Determine $\mathcal{P}[51 < X \leq 71]$.
3. Num sistema de transmissão digital a probabilidade média de erro de bit (BER) é $Q(\sqrt{\gamma/2})$, onde γ é a SNR. Qual o menor valor de γ que garante BER $< 10^{-6}$?

Notes

Distribuição de uma Função de uma Variável Aleatória

- Dado que $Y = g(X) \Rightarrow f_Y(y)$?
  1. Determinar a CDF $F_Y(y) = \mathcal{P}[Y \leq y]$;
  2. Calcular a PDF a partir da derivada $f_Y(y) = dF_Y(y)/dy$.
- **Exemplo 1:** Seja X uma V.A. contínua uniforme $(0{,}1)$. Determinar a PDF de $Y = X^n$.
- **Exemplo 2:** Considere $Y = X^2,\ y \geq 0$. Determine $f_Y(y)$ em função de $f_X(x)$.
- **Exemplo 3:** Conhecendo que $Y = |X|$, determine $f_Y(y)$ em função de $f_X(x)$.

Notes

Distribuição de uma Função de uma Variável Aleatória

Seja X uma variável aleatória contínua com função densidade de probabilidade $f_X(x)$. Suponha que $g(X)$ seja uma função de x estritamente monotônica (crescente ou decrescente) e derivável (portanto contínua). Então a variável aleatória Y definida por $Y = g(X)$ tem uma função densidade de probabilidade dada por

$$f_Y(x) = \begin{cases} f_X[g^{-1}(y)] \left| \dfrac{d}{dy} g^{-1}(y) \right| & \text{se } y = g(x) \text{ para algum } x \\ 0 & \text{se } y \neq g(x) \text{ para todo } x, \end{cases}$$

onde se define $g^{-1}(y)$ de forma que essa função corresponda ao valor de x para o qual $g(x) = y$.

- **Exemplo 4:** Considere $Y = X^n,\ y \geq 0$. Determine $f_Y(y)$ em função de $f_X(x)$.

Notes