Attia, John Okyere. “DC Analysis.”
Electronics and Circuit Analysis using MATLAB.
Ed. John Okyere Attia
Boca Raton: CRC Press LLC, 1999

CHAPTER FOUR

DC ANALYSIS

4.1 NODAL ANALYSIS

Kirchhoff's current law states that for any electrical circuit, the algebraic sum of all the currents at any node in the circuit equals zero. In nodal analysis, if there are n nodes in a circuit, and we select a reference node, the other nodes can be numbered from V_1 through V_{n-1}. With one node selected as the reference node, there will be n-1 independent equations. If we assume that the admittance between nodes i and j is given as Y_{ij}, we can write the nodal equations:

$$\begin{aligned}
Y_{11} V_1 + Y_{12} V_2 + \dots \quad + Y_{1m} V_m &= \sum I_1 \\
Y_{21} V_1 + Y_{22} V_2 + \dots \quad + Y_{2m} V_m &= \sum I_2 \\
Y_{m1} V_1 + Y_{m2} V_2 + \dots \quad + Y_{mm} V_m &= \sum I_m
\end{aligned} \tag{4.1}$$

where

$m = n - 1$

V_1, V_2 and V_m are voltages from nodes 1, 2 and so on ..., n with respect to the reference node.

$\sum I_x$ is the algebraic sum of current sources at node x.

Equation (4.1) can be expressed in matrix form as

$$[Y][V] = [I] \tag{4.2}$$

The solution of the above equation is

$$[V] = [Y]^{-1}[I] \tag{4.3}$$

where

$[Y]^{-1}$ is an inverse of $[Y]$.

In MATLAB, we can compute [V] by using the command

$$V = inv(Y) * I \tag{4.4}$$

where

$inv(Y)$ is the inverse of matrix Y

The matrix left and right divisions can also be used to obtain the nodal voltages. The following MATLAB commands can be used to find the matrix [V]

$$V = {}^{I}\!/_{Y} \tag{4.5}$$

or

$$V = Y \backslash I \tag{4.6}$$

The solutions obtained from Equations (4.4) to (4.6) will be the same, provided the system is not ill-conditioned. The following two examples illustrate the use of MATLAB for solving nodal voltages of electrical circuits.

Example 4.1

For the circuit shown below, find the nodal voltages V_1, V_2 and V_3.

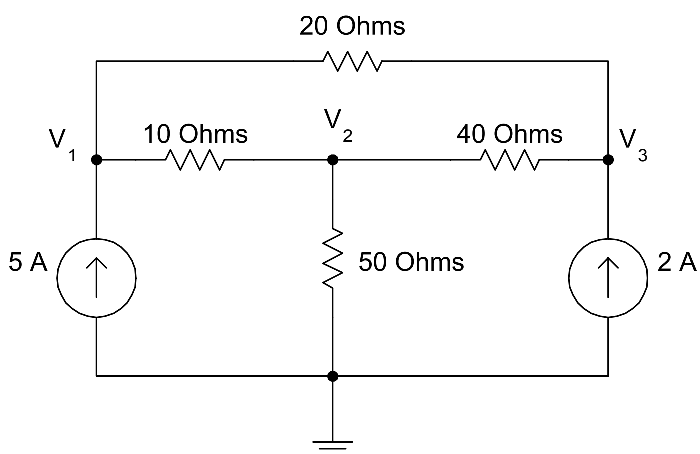

Figure 4.1 Circuit with Nodal Voltages

Solution

Using KCL and assuming that the currents leaving a node are positive, we have

For node 1,

$$\frac{V_1 - V_2}{10} + \frac{V_1 - V_3}{20} - 5 = 0$$

i.e.,

$$0.15V_1 - 0.1V_2 - 0.05V_3 = 5 \tag{4.7}$$

At node 2,

$$\frac{V_2 - V_1}{10} + \frac{V_2}{50} + \frac{V_2 - V_3}{40} = 0$$

i.e.,

$$-0.1V_1 + 0.145V_2 - 0.025V_3 = 0 \tag{4.8}$$

At node 3,

$$\frac{V_3 - V_1}{20} + \frac{V_3 - V_2}{40} - 2 = 0$$

i.e.,

$$-0.05V_1 - 0.025V_2 + 0.075V_3 = 2 \tag{4.9}$$

In matrix form, we have

$$\begin{bmatrix} 0.15 & -0.1 & -0.05 \\ -0.1 & 0.145 & -0.025 \\ -0.05 & -0.025 & 0.075 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix} \tag{4.10}$$

The MATLAB program for solving the nodal voltages is

MATLAB Script

```
diary ex4_1.dat
% program computes the nodal voltages
```

```
% given the admittance matrix Y and current vector I
% Y is the admittance matrix and I is the current vector
% initialize matrix y and vector I using YV=I form
Y = [ 0.15    -0.1     -0.05;
     -0.1      0.145   -0.025;
     -0.05    -0.025    0.075];
I = [5;
     0;
     2];
% solve for the voltage
fprintf('Nodal voltages V1, V2 and V3 are \n')
v = inv(Y)*I
diary
```

The results obtained from MATLAB are

```
Nodal voltages V1, V2 and V3,

v =
      404.2857
      350.0000
      412.8571
```

Example 4.2:

Find the nodal voltages of the circuit shown below.

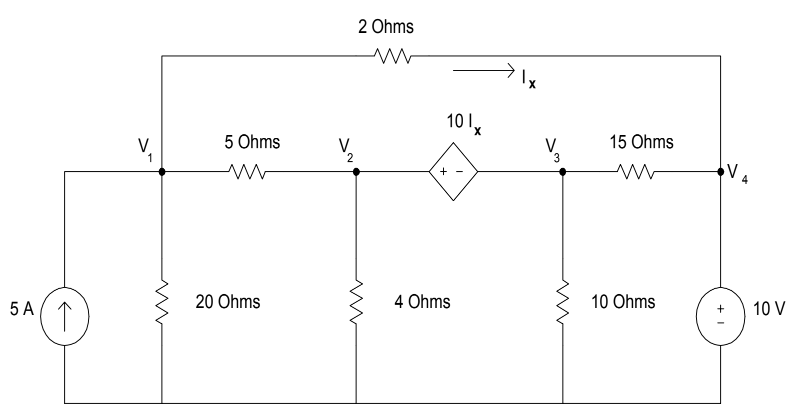

Figure 4.2 Circuit with Dependent and Independent Sources

Solution

Using KCL and the convention that currents leaving a node is positive, we have

At node 1

$$\frac{V_1}{20} + \frac{V_1 - V_2}{5} + \frac{V_1 - V_4}{2} - 5 = 0$$

Simplifying, we get

$$0.75V_1 - 0.2V_2 - 0.5V_4 = 5 \qquad (4.11)$$

At node 2,

$$V_2 - V_3 = 10I_X$$

But

$$I_X = \frac{(V_1 - V_4)}{2}$$

Thus

$$V_2 - V_3 = \frac{10(V_1 - V_4)}{2}$$

Simplifying, we get

$$-5V_1 + V_2 - V_3 + 5V_4 = 0 \qquad (4.12)$$

From supernodes 2 and 3, we have

$$\frac{V_3}{10} + \frac{V_2 - V_1}{5} + \frac{V_2}{4} + \frac{V_3 - V_4}{15} = 0$$

Simplifying, we get

$$-0.2V_1 + 0.45V_2 + 0.1667V_3 - 0.06667V_4 = 0 \qquad (4.13)$$

At node 4, we have

$$V_4 = 10 \tag{4.14}$$

In matrix form, equations (4.11) to (4.14) become

$$\begin{bmatrix} 0.75 & -0.2 & 0 & -0.5 \\ -5 & 1 & -1 & 5 \\ -0.2 & 0.45 & 0.1667 & -0.06667 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \\ 10 \end{bmatrix} \tag{4.15}$$

The MATLAB program for solving the nodal voltages is

MATLAB Script

```
diary ex4_2.dat
% this program computes the nodal voltages
% given the admittance matrix Y and current vector I
% Y is the admittance matrix
% I is the current vector
% initialize the matrix y and vector I using YV=I

Y = [0.75   -0.2   0   -0.5;
       -5      1  -1      5;
      -0.2   0.45  0.166666667   -0.0666666667;
       0      0   0      1];

% current vector is entered as a transpose of row vector
I = [5   0   0   10]';

% solve for nodal voltage
fprintf('Nodal voltages V1,V2,V3,V4 are \n')
V = inv(Y)*I
diary
```

We obtain the following results.

```
Nodal voltages V1,V2,V3,V4 are
```

V =
18.1107
17.9153
-22.6384
10.0000

4.2 LOOP ANALYSIS

Loop analysis is a method for obtaining loop currents. The technique uses Kirchoff voltage law (KVL) to write a set of independent simultaneous equations. The Kirchoff voltage law states that the algebraic sum of all the voltages around any closed path in a circuit equals zero.

In loop analysis, we want to obtain current from a set of simultaneous equations. The latter equations are easily set up if the circuit can be drawn in planar fashion. This implies that a set of simultaneous equations can be obtained if the circuit can be redrawn without crossovers.

For a planar circuit with n-meshes, the KVL can be used to write equations for each mesh that does not contain a dependent or independent current source. Using KVL and writing equations for each mesh, the resulting equations will have the general form:

$$Z_{11}I_1 + Z_{12} I_2 + Z_{13} I_3 + \quad \ldots \quad Z_{1n} I_n = \sum V_1$$

$$Z_{21} I_1 + Z_{22} I_2 + Z_{23} I_3 + \quad \ldots \quad Z_{2n} I_n = \sum V_2$$

$$Z_{n1} I_1 + Z_{n2} I_2 + Z_{n3} I_3 + \quad \ldots \quad Z_{nn} I_n = \sum V_n \tag{4.16}$$

where

$I_1, I_2, \ldots I_n$ are the unknown currents for meshes 1 through n.

$Z_{11}, Z_{22}, \ldots, Z_{nn}$ are the impedance for each mesh through which individual current flows.

$Z_{ij}, \quad j \# i$ denote mutual impedance.

$\sum V_x$ is the algebraic sum of the voltage sources in mesh x.

Equation (4.16) can be expressed in matrix form as

$$[Z][I] = [V] \tag{4.17}$$

where

$$Z = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & \dots & Z_{1n} \\ Z_{21} & Z_{22} & Z_{23} & \dots & Z_{2n} \\ Z_{31} & Z_{32} & Z_{33} & \dots & Z_{3n} \\ .. & .. & . & \dots & .. \\ Z_{n1} & Z_{n2} & Z_{n3} & \dots & Z_{nn} \end{bmatrix}$$

$$I = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ . \\ I_n \end{bmatrix}$$

and

$$V = \begin{bmatrix} \sum V_1 \\ \sum V_2 \\ \sum V_3 \\ .. \\ \sum V_n \end{bmatrix}$$

The solution to Equation (4.17) is

$$[I] = [Z]^{-1}[V] \tag{4.18}$$

In MATLAB, we can compute [I] by using the command

$$I = inv(Z) * V \tag{4.19}$$

where

$inv(Z)$ is the inverse of the matrix Z

The matrix left and right divisions can also be used to obtain the loop currents. Thus, the current I can be obtained by the MATLAB commands

$$I = {}^{V}\!/_{Z} \tag{4.20}$$

or

$$I = Z \backslash V \tag{4.21}$$

As mentioned earlier, Equations (4.19) to (4.21) will give the same results, provided the circuit is not ill-conditioned. The following examples illustrate the use of MATLAB for loop analysis.

Example 4.3

Use the mesh analysis to find the current flowing through the resistor R_B. In addition, find the power supplied by the 10-volt voltage source.

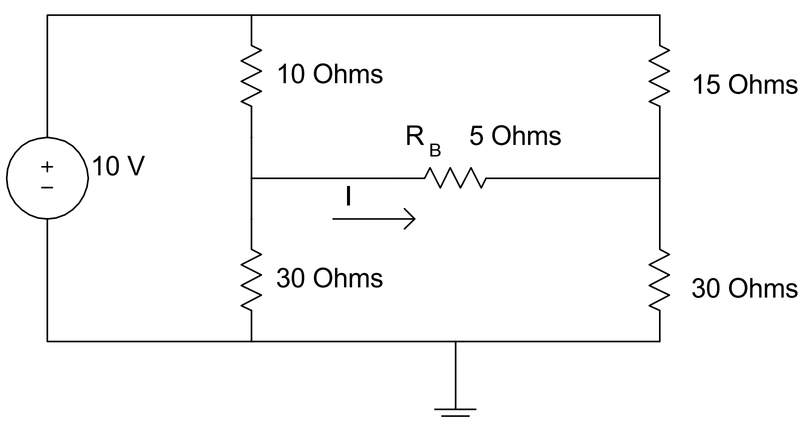

Figure 4.3a Bridge Circuit

Solution

Using loop analysis and designating the loop currents as I_1, I_2, I_3, we obtain the following figure.

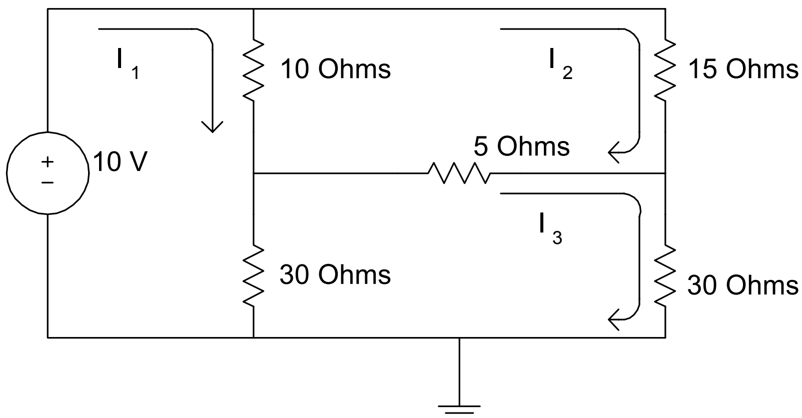

Figure 4.3b Bridge Circuit with Loop Currents

Note that $I = I_3 - I_2$ and power supplied by the source is $P = 10I_1$

The loop equations are

Loop 1,

$$10(I_1 - I_2) + 30(I_1 - I_3) - 10 = 0$$

$$40I_1 - 10I_2 - 30I_3 = 10 \tag{4.22}$$

Loop 2,

$$10(I_2 - I_1) + 15I_2 + 5(I_2 - I_3) = 0$$

$$-10I_1 + 30I_2 - 5I_3 = 0 \tag{4.23}$$

Loop 3,

$$30(I_3 - I_1) + 5(I_3 - I_2) + 30I_3 = 0$$

$$-30I_1 - 5I_2 + 65I_3 = 0 \tag{4.24}$$

In matrix form, Equations (4.22) and (4.23) become

$$\begin{bmatrix} 40 & -10 & -30 \\ -10 & 30 & -5 \\ -30 & -5 & 65 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} \tag{4.25}$$

The MATLAB program for solving the loop currents I_1, I_2, I_3, the current I and the power supplied by the 10-volt source is

MATLAB Script

```
diary ex4_3.dat
% this program determines the current
% flowing in a resistor RB and power supplied  by source
% it computes the loop currents given the impedance
% matrix Z and voltage vector V
% Z is the impedance matrix
% V is the voltage matrix
% initialize the matrix Z and vector V

Z = [40   -10   -30;
    -10    30    -5;
    -30    -5    65];

V = [10  0  0]';

% solve for the loop currents
I = inv(Z)*V;
% current through RB is calculated
IRB = I(3) - I(2);
fprintf('the current through R is %8.3f Amps \n',IRB)
% the power supplied by source is calculated
PS = I(1)*10;
fprintf('the power supplied by 10V source is %8.4f watts \n',PS)
diary
```

MATLAB answers are

```
the current through R is    0.037 Amps
the power supplied by 10V source is   4.7531 watts
```

Circuits with dependent voltage sources can be analyzed in a manner similar to that of example 4.3. Example 4.4 illustrates the use of KVL and MATLAB to solve loop currents.

Example 4.4

Find the power dissipated by the 8 Ohm resistor and the current supplied by the 10-volt source.

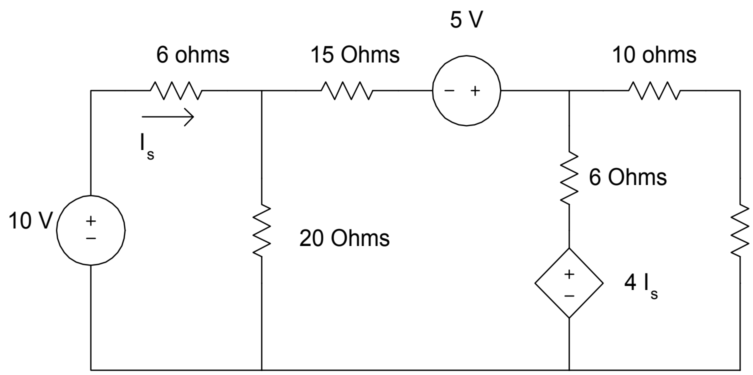

Figure 4.4a Circuit for Example 4.4

Solution

Using loop analysis and denoting the loop currents as I_1, I_2 and I_3, the circuit can be redrawn as

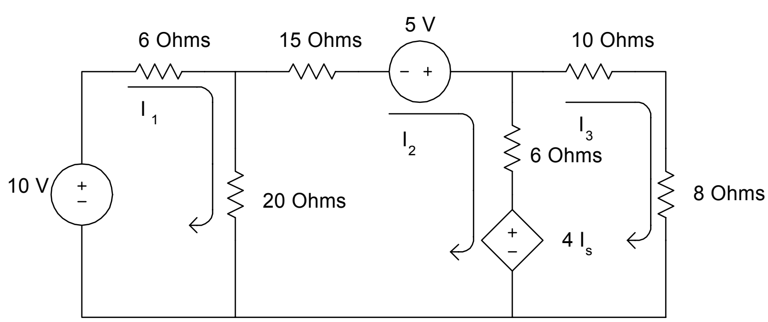

Figure 4.4b Figure 4.4 with Loop Currents

By inspection,

$$I_S = I_1 \tag{4.26}$$

For loop 1,

$$-10 + 6I_1 + 20(I_1 - I_2) = 0$$

$$26I_1 - 20I_2 = 10 \tag{4.27}$$

For loop 2,

$$15I_2 - 5 + 6(I_2 - I_3) + 4I_S + 20(I_2 - I_1) = 0$$

Using Equation (4.26), the above expression simplifies to

$$-16I_1 + 41I_2 - 6_3 I = 5 \tag{4.28}$$

For loop 3,

$$10I_3 + 8I_3 - 4I_S + 6(I_3 - I_2) = 0$$

Using Equation (4.26), the above expression simplifies to

$$-4I_1 - 6I_2 + 24I_3 = 0 \tag{4.29}$$

Equations (4.25) to (4.27) can be expressed in matrix form as

$$\begin{bmatrix} 26 & -20 & 0 \\ -16 & 41 & -6 \\ -4 & -6 & 24 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 0 \end{bmatrix} \tag{4.30}$$

The power dissipated by the 8 Ohm resistor is

$$P = RI_3^2 = 8I_3^2$$

The current supplied by the source is $I_S = I_1$

A MATLAB program for obtaining the power dissipated by the 8 Ohm resistor and the current supplied by the source is shown below

MATLAB Script

```
diary ex4_4.dat
% This program determines the power dissipated by
% 8 ohm resistor and current supplied by the
% 10V source
%
% the program computes the loop currents, given
% the impedance matrix Z and voltage vector V
%
% Z is the impedance matrix
% V is the voltage vector
% initialize the matrix Z and vector V of equation
% ZI=V

Z = [26    -20    0;
    -16     40   -6;
     -4     -6   24];
V = [10   5   0]';

% solve for loop currents
I = inv(Z)*V;
% the power dissipation in 8 ohm resistor is P
P = 8*I(3)^2;
% print out the results
fprintf('Power dissipated in 8 ohm resistor is %8.2f Watts\n',P)
fprintf('Current in 10V source is %8.2f Amps\n',I(1))
diary
```

MATLAB results are

```
Power dissipated in 8 ohm resistor is     0.42  Watts
Current in 10V source is     0.72 Amps
```

For circuits that contain both current and voltage sources, irrespective of whether they are dependent sources, both KVL and KVL can be used to obtain equations that can be solved using MATLAB. Example 4.5 illustrates one such circuit.

Example 4.5

Find the nodal voltages in the circuit, i.e., $V_1, V_2, \ldots, V_5$

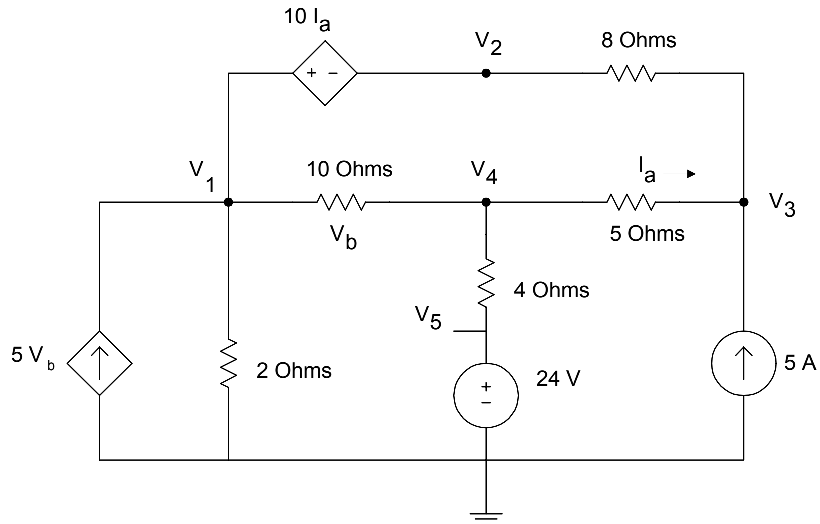

Figure 4.5 Circuit for Example 4.5

Solution

By inspection,

$$V_b = V_1 - V_4 \tag{4.31}$$

Using Ohm's Law

$$I_a = \frac{V_4 - V_3}{5} \tag{4.32}$$

Using KCL at node 1, and supernode 1-2, we get

$$\frac{V_1}{2} + \frac{V_1 - V_4}{10} - 5V_b + \frac{V_2 - V_3}{8} = 0 \tag{4.33}$$

Using Equation (4.31), Equation (4.33) simplifies to

$$-4.4V_1 + 0.125V_2 - 0.125V_3 + 4.9V_4 = 0 \tag{4.34}$$

Using KCL at node 4, we have

$$\frac{V_4 - V_5}{4} + \frac{V_4 - V_3}{5} + \frac{V_4 - V_1}{10} = 10$$

This simplifies to

$$-0.1V_1 - 0.2V_3 + 0.55V_4 - 0.25V_5 = 0 \tag{4.35}$$

Using KCL at node 3, we get

$$\frac{V_3 - V_4}{5} + \frac{V_3 - V_2}{8} - 5 = 0$$

which simplifies to

$$-0.125V_2 + 0.325V_3 - 0.2V_4 = 5 \tag{4.36}$$

Using KVL for loop 1, we have

$$-10I_a + V_b + 5I_a + 8(I_a + 5) = 0 \tag{4.37}$$

Using Equations (4.31) and (4.32), Equation (4.37) becomes

$$-10I_a + V_b + 5I_a + 8I_a + 40 = 0$$

i.e.,

$$3I_a + V_b = -40$$

Using Equation (4.32), the above expression simplifies to

$$3\frac{V_4 - V_3}{5} + V_1 - V_4 = -40$$

Simplifying the above expression, we get

$$V_1 - 0.6V_3 - 0.4V_4 = -40 \tag{4.38}$$

By inspection

$$V_S = 24 \tag{4.39}$$

Using Equations (4.34), (4.35), (4.36), (4.38) and (4.39), we get the matrix equation

$$\begin{bmatrix} -4.4 & 0.125 & -0.125 & 4.9 & 0 \\ -0.1 & -0.2 & 0 & 0.55 & -0.25 \\ 0 & -0.125 & 0.325 & -0.2 & 0 \\ 1 & 0 & -0.6 & -0.4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 5 \\ -40 \\ 24 \end{bmatrix} \qquad (4.40)$$

The MATLAB program for obtaining the nodal voltages is shown below.

MATLAB Script

```
diary ex4_5.dat
% Program determines the nodal voltages
%          given an admittance matrix Y and  current vector I
% Initialize matrix Y and the current vector I of
%          matrix equation Y V = I
Y = [-4.4  0.125  -0.125  4.9  0;
    -0.1  0      -0.2    0.55  -0.25;
     0    -0.125  0.325  -0.2   0;
     1    0      -0.6    -0.4   0;
     0    0       0       0     1];
I = [0  0  5  -40  24]';
%  Solve for the nodal voltages
fprintf('Nodal voltages V(1), V(2), .. V(5) are \n')
V = inv(Y)*I; diary
```

The results obtained from MATLAB are

```
Nodal voltages V(1), V(2), ... V(5) are

V =
  117.4792
  299.7708
  193.9375
  102.7917
   24.0000
```

4.3 MAXIMUM POWER TRANSFER

Assume that we have a voltage source V_S with resistance R_S connected to a load R_L. The circuit is shown in Figure 4.6.

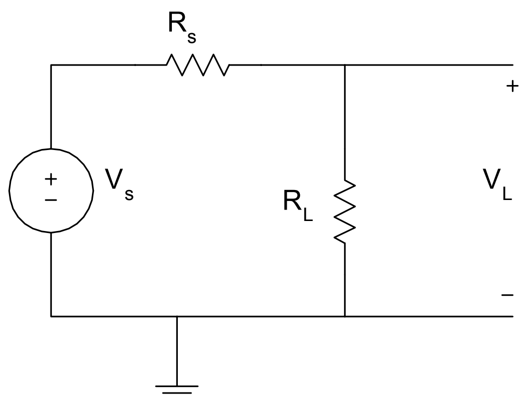

Figure 4.6 Circuit for Obtaining Maximum Power Dissipation

The voltage across the Load R_L is given as

$$V_L = \frac{V_s R_L}{R_s + R_L}$$

The power dissipated by the load R_L is given as

$$P_L = \frac{V_L^2}{R_L} = \frac{V_s^2 R_L}{(R_s + R_L)^2} \tag{4.41}$$

The value of R_L that dissipates the maximum power is obtained by differentiating P_L with respect to R_L, and equating the derivative to zero. That is,

$$\begin{aligned} \frac{dP_L}{dR_L} &= \frac{(R_s + R_L)^2 V_S - V_s^2 R_L (2)(R_s + R_L)}{(R_s + R_L)^4} \\ \frac{dP_L}{dR_L} &= 0 \end{aligned} \tag{4.42}$$

Simplifying the above we get

$$(R_s + R_L) - 2R_L = 0$$

i.e.,

$$R_L = R_S \tag{4.43}$$

Thus, for a resistive network, the maximum power is supplied to a load provided the load resistance is equal to the source resistance. When R_L = 0, the voltage across and power dissipated by R_L are zero. On the other hand, when R_L approaches infinity, the voltage across the load is maximum, but the power dissipation is zero. MATLAB can be used to observe the voltage across and power dissipation of the load as functions of load resistance value. Example 4.6 shows the use of MATLAB to plot the voltage and display the power dissipation of a resistive circuit.

Before presenting an example on the maximum power transfer theorem, let us discuss the MATLAB functions **diff** and **find**.

4.3.1 MATLAB Diff and Find Functions

Numerical differentiation can be obtained using the backward difference expression

$$f'(x_n) = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}} \tag{4.44}$$

or by the forward difference expression

$$f'(x_n) = \frac{f(x_{n+1}) - f(x_n)}{x_{n+1} - x_n} \tag{4.45}$$

The derivative of $f(x)$ can be obtained by using the MATLAB **diff** function as

$$f'(x) \cong diff(f)./diff(x). \tag{4.46}$$

If f is a row or column vector

$$f = [f(1) \quad f(2) \quad \ldots \quad f(n)]$$

then **diff(f)** returns a vector of difference between adjacent elements

$$diff(f) = [f(2) - f(1) \quad f(3) - f(2) \quad \ldots \quad f(n) - f(n-1)] \tag{4.47}$$

The output vector $diff(f)$ will be one element less than the input vector f.

The **find** function determines the indices of the nonzero elements of a vector or matrix. The statement

B = find(f) (4.48)

will return the indices of the vector f that are nonzero. For example, to obtain the points where a change in sign occurs, the statement

Pt_change = find(product < 0) (4.49)

will show the indices of the locations in *product* that are negative.

The **diff** and **find** are used in the following example to find the value of resistance at which the maximum power transfer occurs.

Example 4.6

In Figure 4.7, as R_L varies from 0 to 50KΩ, plot the power dissipated by the load. Verify that the maximum power dissipation by the load occurs when R_L is 10 KΩ.

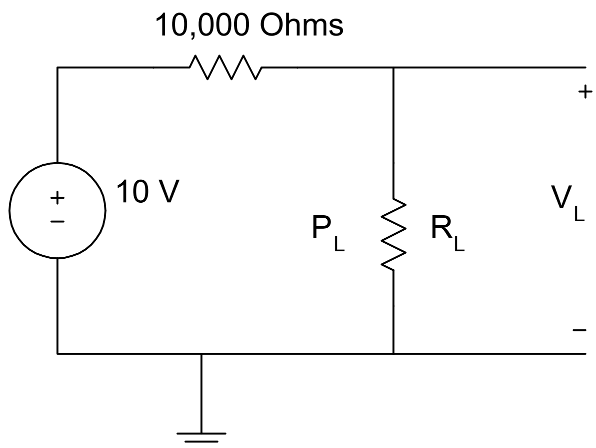

Figure 4.7 Resistive Circuit for Example 4.6

Solution

MATLAB Script

```
% maximum power transfer
% vs is the supply voltage
% rs is the supply resistance
% rl is the load resistance
% vl is the voltage across the load
% pl is the power dissipated by the load
vs = 10;    rs = 10e3;
rl = 0:1e3:50e3;
k = length(rl); % components in vector rl
% Power dissipation calculation
for i=1:k
  pl(i) = ((vs/(rs+rl(i)))^2)*rl(i);
end
% Derivative of power is calculated using backward difference
dp = diff(pl)./diff(rl);
rld = rl(2:length(rl)); % length of rld is 1 less than that of rl
% Determination of critical points of derivative of power
prod = dp(1:length(dp) - 1).*dp(2:length(dp));
crit_pt = rld(find(prod < 0));
max_power = max(pl); % maximum power is calculated
% print out results
```

```
fprintf('Maximum power occurs at %8.2f Ohms\n',crit_pt)
fprintf('Maximum power dissipation is %8.4f Watts\n', max_power)
% Plot power versus load
plot(rl,pl,'+')
title('Power delivered to load')
xlabel('load resistance in Ohms')
ylabel('power in watts')
```

The results obtained from MATLAB are

```
Maximum power occurs at 10000.00 Ohms
Maximum power dissipation is   0.0025 Watts
```

The plot of the power dissipation obtained from MATLAB is shown in Figure 4.8.

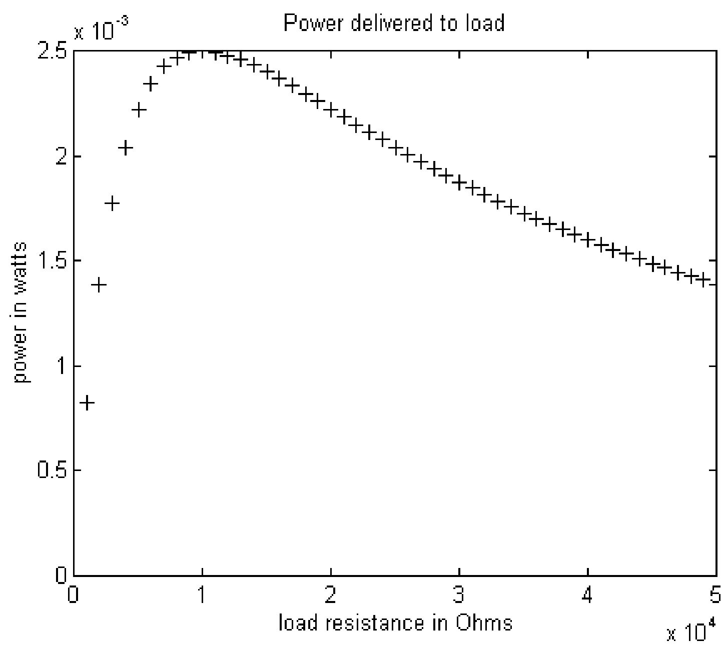

Figure 4.8 Power delivered to load

SELECTED BIBLIOGRAPHY

1. MathWorks, Inc., *MATLAB, High-Performance Numeric Computation Software*, 1995.

2. Etter, D.M., *Engineering Problem Solving with MATLAB*, 2nd Edition, Prentice Hall, 1997.

3. Gottling, J.G., *Matrix Analysis of Circuits Using MATLAB*, Prentice Hall, 1995.

4. Johnson, D.E., Johnson, J.R. and Hilburn, J.L., *Electric Circuit Analysis*, 3rd Edition, Prentice Hall, 1997.

5. Dorf, R.C. and Svoboda, J.A., *Introduction to Electric Circuits*, 3rd Edition, John Wiley & Sons, 1996.

EXERCISES

4.1 Use loop analysis to write equations for the circuit shown in Figure P4.1. Determine the current I using MATLAB.

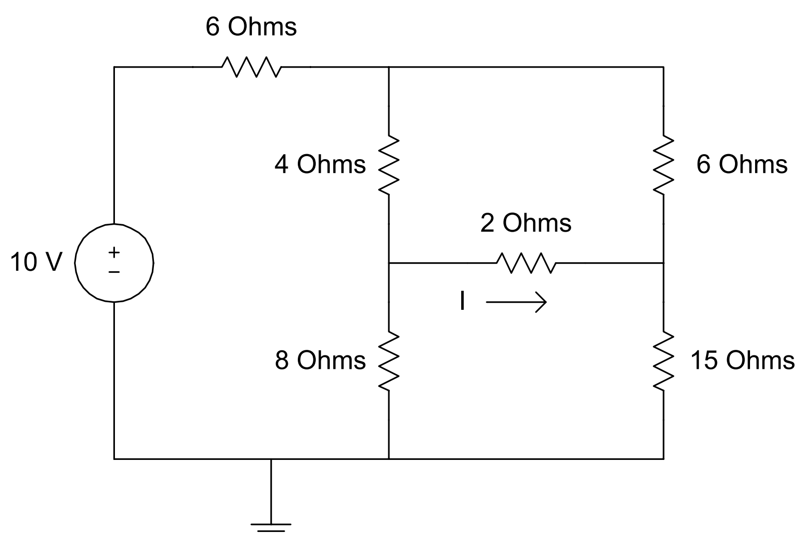

Figure P4.1 Circuit for Exercise 4.1

4.2 Use nodal analysis to solve for the nodal voltages for the circuit shown in Figure P4.2. Solve the equations using MATLAB.

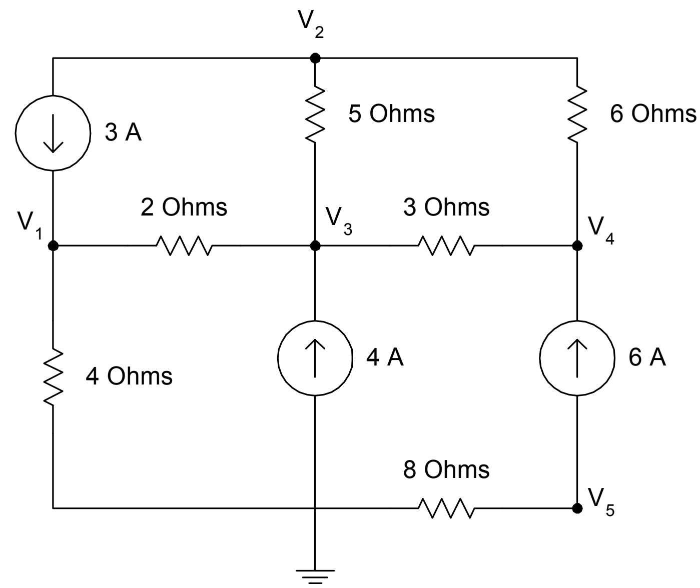

Figure P4.2 Circuit for Exercise 4.2

4.3 Find the power dissipated by the 4Ω resistor and the voltage V_1.

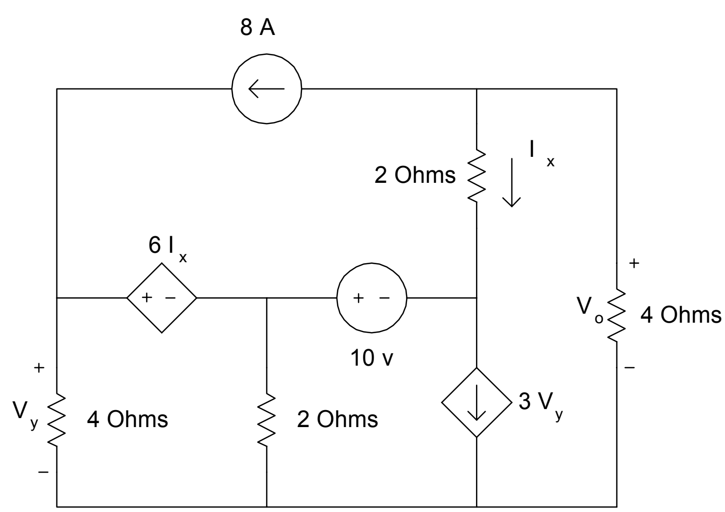

Figure P4.3 Circuit for Exercise 4.3

4.4 Using both loop and nodal analysis, find the power delivered by a 15V source.

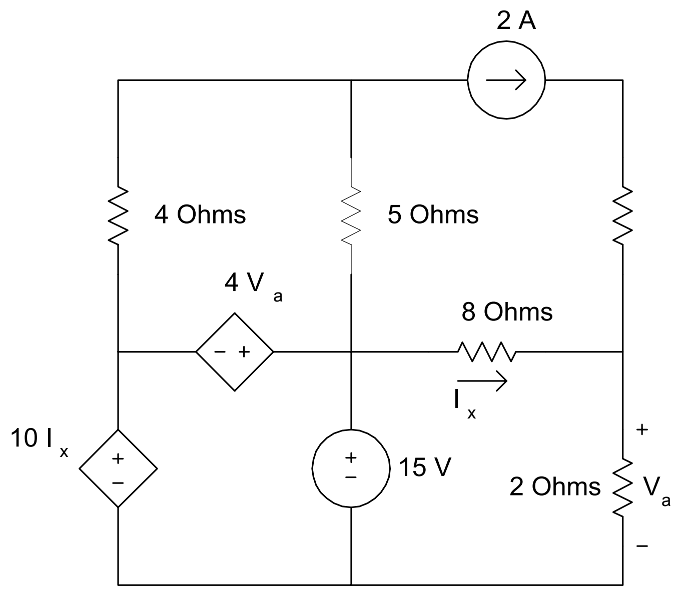

Figure P4.4 Circuit for Exercise 4.4

4.5 As R_L varies from 0 to 12 in increments of 2Ω, calculate the power dissipated by R_L. Plot the power dissipation with respect to the variation in R_L. What is the maximum power dissipated by R_L? What is the value of R_L needed for maximum power dissipation?

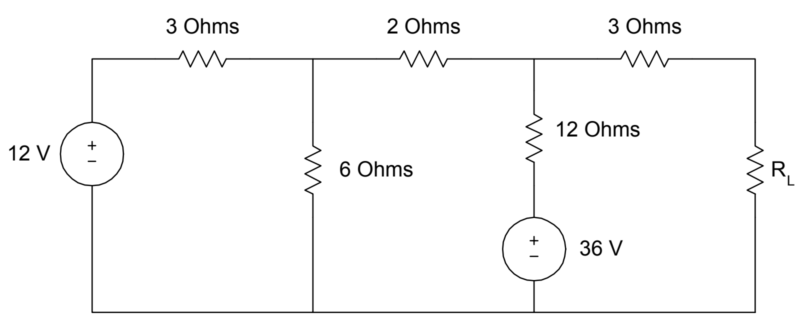

Figure P4.5 Circuit for Exercise 4.5

4.6 Using loop analysis and MATLAB, find the loop currents. What is the power supplied by the source?

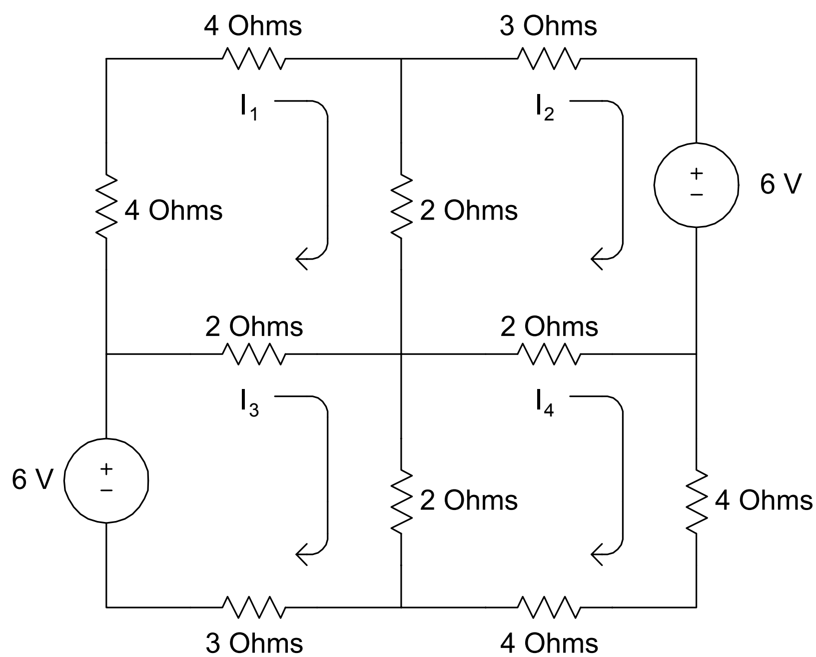

Figure P4.6 Circuit for Exercise 4.6