Power Flow Analysis for Interconnected T&D Networks with Meshed Topology

C. C. Durce, E. M. Lourenço, and O. L. Tortelli

Abstract—This paper proposes an extension of the conventional power flow formulation in order to enable it to process interconnected transmission and distribution (T&D) networks with meshed topologies. The proposed extension applies a complex per unit normalization to adequate X/R ratios of Distribution Feeders (DF) to the same levels of Transmission Systems. Additionally, active and reactive power compensations are included at each T&D boundary bus and treated as new state variables, so that the power flow solution of the normalized system is the same as the original network. Appropriate equivalence between power flows through branches connected to boundary buses is included into the problem as new equations, resulting in a solvable non-redundant set of algebraic equations. The impact of these modifications on the power flow solution via Newton-Raphson’s method is also presented and discussed. The proposed approach provides an efficient tool to perform power flow analysis on T&D systems, where DFs are explicitly represented, allowing an accurate determination of their impact onto the transmission operation. Besides, meshed topologies can be easily handled by means of the proposed approach. Simulation results conducted on an illustrative test-system are shown in the paper. Distinct operational conditions between T&D parts of the test-system allow both the evaluation of the features and the performance of the proposed approach.

Index Terms—Power Flow, Transmission and Distribution Systems, Meshed Topology, Distributed Generation, Complex Per Unit Normalization.

I. INTRODUCTION

CONVENCIONAL Power Flow methodologies have been designed considering the partition of the system into transmission and distribution networks. This procedure allowed the development of different techniques of power flow solution to be applied for each partition, according to its specific characteristics.

Transmission Systems (TS) present high voltage levels, meshed topology and high X/R ratio which lead to the well-known Pθ–QV decoupling. The load flow problem is usually solved by the conventional Newton-Raphson’s method and its fast decoupled versions [1-2].

For Distribution Systems, however, where low levels of voltage and X/R ratio and radial operation are the typical characteristics, the back-forward sweep families of algorithms are usually employed [3-5].

However, the dissemination of Distributed Generation (DG) and the advent of Smart Grid technologies have increased the influence of DFs on the operation of TSs. The traditional closed ring operation of DFs, for example, has been reinforced in this current context [6]. A forward-looking DS indicates that meshed topology would be significantly relevant, increasing the operation reliability and, consequently, improving the energy supply [7]. It is also important to notice that although DSs is moving towards a topology similar to the TSs, including the presence of generation units, the networks parameters tend to preserve their original characteristics. That is, different levels of X/R ratios are still faced by interconnected T&D systems, jeopardizing the direct application of Newton’s based algorithms, such as fast decoupled versions.

The above discussion points out that the application of DS’s power flow algorithms, which are based on radial (or even weakly meshed) topologies, are increasingly limited. Also, despite of previous efforts presented in the literature [8-11], the solution of power flow analysis for interconnected T&D system considering the new context of Smart Grids is still a challenge.

The objective of this paper is to propose an extension of the conventional power flow formulation via Newton-Raphson’s method in order to accommodate the explicit representation of distribution feeders on the network model. The method is numerically robust and yields to the power flow solution of the whole interconnected T&D power network, which allows an accurate evaluation of the interference of DSs onto TS’s operation.

The proposed approach builds on previous efforts related to the use of axis rotation on the fast decoupled Newton-Raphson’s method [12-14]. The concepts introduced in these references provide the foundations to the Complex Per Unit Normalization (c.p.u) proposed in [6].

The power flow approach proposed in this paper is accomplished by applying c.p.u. to adjust X/R ratios of explicitly represented distribution feeders to the same levels of transmission systems. Additionally, to preserve the original network solution, fictitious active and reactive power compensation are included at each T&D boundary bus.

This paper provides details on how the required changes should be embedded in the conventional power flow formulation. A small test-system is employed to both illustrate
the features and validate the results obtained with the proposed methodology.

II. PREVIOUS APPROACH

The approach proposed in this paper is based on the complex per unit normalization presented in the previous ISGT Europe Conference [6]. This section presents the foundations of this approach so that the improvements of the new method, presented in Section III, can be thoroughly understood.

A. Complex Per Unit Normalization: c.p.u.

Complex per unit normalization can be seen as a different interpretation of the axis rotation techniques [12-14]. The former is based on the well-known per unit (p.u.) normalization of power systems, as discussed next.

Differently from p.u. normalization, where power base is a real value, c.p.u. is based on the adoption of a complex volt-ampere base, that is:

\[
\hat{Z}_{\text{base}} = \frac{V_{\text{base}}^2}{S_{\text{base}}} = \frac{Z_{\text{base}}}{e^{-j\phi_{\text{base}}}}
\]  

(1)

where \(\phi_{\text{base}}\) is the volt-ampere base angle.

However, voltage bases are kept real and defined just as in the conventional p.u. normalization, that is, a different (but related) magnitude value is adopted for each voltage level. Therefore, the impedance base is defined as:

\[
\hat{Z}_{\text{cpu}} = \frac{V_{\text{cpu}}^2}{S_{\text{cpu}}} = \frac{Z_{\text{cpu}}}{e^{-j\phi_{\text{cpu}}}}
\]  

(2)

According to equation (2), the impedance magnitude provided by the c.p.u. normalization has the same value as that of the conventional approach. However, the impedance angle is normalized by the volt-ampere base angle, that is:

\[
\hat{Z}_{\text{cpu}} = \frac{R_{\text{cpu}} + jX_{\text{cpu}}}{Z_{\text{base}}} = \frac{Z_{\text{cpu}}}{Z_{\text{base}}} \cdot e^{-j\phi_{\text{base}}} = (Z_{\text{cpu}} \cdot e^{-j\phi_{\text{base}}}) e^{j\phi_{\text{base}}}
\]  

(3)

where \(\theta\) is the original phase angle of the impedance.

From Eq. (3), one concludes that:

\[
R_{\text{cpu}} = R_{\text{pu}} \cdot \cos \phi_{\text{base}} - X_{\text{pu}} \sin \phi_{\text{base}}
\]  

(4)

\[
X_{\text{cpu}} = X_{\text{pu}} \cdot \cos \phi_{\text{base}} + R_{\text{pu}} \sin \phi_{\text{base}}
\]  

(5)

Therefore, X/R ratio is given by:

\[
\frac{X_{\text{cpu}}}{R_{\text{cpu}}} = \tan(\theta + \phi_{\text{base}})
\]  

(6)

Finally, active and reactive power injections must be properly normalized by the complex volt-ampere base, that is:

\[
\hat{S}_{\text{cpu}} = \frac{S_{\text{cpu}}}{S_{\text{base}}} \cdot e^{j\delta_{\text{base}}} = \frac{P_{\text{pu}} + jQ_{\text{pu}}}{e^{j\phi_{\text{base}}}}
\]  

(7)

where \(\delta\) is the original phase angle of the volt-ampere power.

Also, from Eq. (7):

\[
P_{\text{cpu}} = P_{\text{pu}} \cdot \cos \phi_{\text{base}} - Q_{\text{pu}} \sin \phi_{\text{base}}
\]  

(8)

\[
Q_{\text{cpu}} = Q_{\text{pu}} \cdot \cos \phi_{\text{base}} + P_{\text{pu}} \sin \phi_{\text{base}}
\]  

(9)

The main advantage of such a complex normalization is that X/R ratios can be adjusted by simply adopting an adequate value for the base angle, \(\phi_{\text{base}}\), as shown in Eq. (6).

B. Power Flow Analysis with c.p.u.

The complex normalization presented in the previous section is applied in [6] to allow the application of the fast decoupled power flow method to DSs. The base angle is defined to each DF according to its distribution lines parameters. Since the base for voltage magnitude is kept real, the base angle for impedance and power are the same, as stated in equations (1) and (2). Consequently, the power flow solution obtained with the complex normalized system is exactly the same as the one obtained with the conventional p.u. normalization.

Furthermore, a first incursion was made in [6] regarding the solution of power flow for interconnected T&D systems. In this case, a different base angle can be applied to each feeder, if needed, while a null base angle is adopted for the TS. This procedure ensures adequate X/R ratios for the whole T&D system. However, to preserve the equivalence between the normalized and the original network, active and reactive power injections at T&D boundary buses are adjusted as follow [6]:

\[
P_k = P_{k}^{\text{pu}} - \sum_{i=1}^{nF} P_{k,i}^{\text{pu}}
\]  

(10)

\[
Q_k = Q_{k}^{\text{pu}} - \sum_{i=1}^{nF} Q_{k,i}^{\text{pu}}
\]  

(11)

where:

\(P_k\), \(Q_k\): new active and reactive power injection at boundary bus \(k\), respectively;

\(P_{k}^{\text{pu}}\), \(Q_{k}^{\text{pu}}\): p.u. values of active and reactive power injections at boundary bus \(k\) when DFs are not explicitly represented, respectively;

\(P_{k,i}^{\text{pu}}\), \(Q_{k,i}^{\text{pu}}\): c.p.u. values of active and reactive power injections required by feeder \(i\), respectively;

\(nF\): number of DFs which are explicitly represented.

The above methodology allows the application of fast decoupled power flow to interconnected T&D systems when the DFs are connected radially to the TS. It means that the method is able to deal with weekly meshed topologies, as illustrated in Fig. 1.
However, this approach assumes that the active and reactive power required by each DF at the boundary bus is previously known, as stated in Eqs. (10) and (11). This assumption imposes the following limitations:

(a) Power flow solution is not accurate, since effective injection at boundary buses depends on the system operational condition;

(b) It cannot be applied when a closed ring operation occurs among DFs which are connected to the TS through distinct boundary buses, as illustrated in Fig. 2. That is, although a weakly meshed topology is allowed for the DS, the meshed topology is still a limitation of the method.

The new methodology proposed in this paper circumvents the above limitations, as described in the following section.

III. NEW APPROACH: EXTENDED POWER FLOW

In this section, the conventional power flow formulation is extended to make it suitable for T&D networks. This extension is motivated by the trend towards the new context raised by the Smart Grid technologies and DG sources dissemination, which emphasizes the needs for pliable tools to deal with any part of an interconnected T&D system.

The c.p.u. normalization presented in Section II.A is used to adjust X/R ratios of the DF that have been selected to be explicitly represented in the network model. The assignment of adequate base angle to each feeder ensures that X/R ratios of the whole system are equivalent to traditional TS levels.

Differently from [6], however, the method proposed in this paper does not assume that power injections required by DFs are known in advance. Instead, discrepancies in power injections, caused by the use of different base angles in the c.p.u. are treated through the inclusion of fictitious power compensation at each boundary bus. The active and reactive power compensations are modeled as new state variables along with complex voltage at network buses. In addition, the relationships between active and reactive power flows through the branches adjacent to each boundary bus are included into the power flow problem as new equations, producing a solvable non-redundant set of algebraic equations, as described below.

A. Power Compensation at Boundary Buses

As mentioned above, the state vector is extended to include the active and reactive power compensation at each boundary bus. Thus, the compensation at boundary bus $k$, illustrated in Fig. 3, is modeled by treating its active and reactive power, $C_k^P$ and $C_k^Q$, as new state variables. This result in an extended state vector, $\mathbf{x}$, given by:

$$\mathbf{x} = [\theta \ V \ C^P \ C^Q]^T$$

where $C^P$, $C^Q$ are $(N_{bd} \times 1)$ vectors of active and reactive power compensation, respectively, and $N_{bd}$ is the number of boundary buses.

Accordingly, expressions for power injections at boundary buses must be modified to comply with the changes on the set of state variables. Thus, the active and reactive power injection at a boundary bus $k$ can be computed by:

$$P_k = C_k^P + V_k \sum_{m \in K} V_m (G_{km} \cos \theta_{km} + B_{km} \sin \theta_{km})$$

$$Q_k = C_k^Q + V_k \sum_{m \in K} V_m (G_{km} \sin \theta_{km} - B_{km} \cos \theta_{km})$$

where $K$ is the set of buses adjacent to bus $k$ including bus $k$.

Simultaneously, the information regarding the relationship between the power flows through distribution lines connected to the boundary bus and the power flow through the boundary transformer (converted to the same base of the DF) is included into the power flow formulation as new sets of equations to be solved along with the network equations. Hence, if the boundary transformer is connected through nodes $k$ and $l$, where $k$ is the boundary bus, and $N$ distribution lines are connected at bus $k$, as illustrated in Fig. 3, the following equivalences apply:

$$P_{B_k} = P_{kl}^{new}(V, \theta) + \sum_{m \in \Omega} P_{km}(V, \theta) = 0$$

$$Q_{B_k} = Q_{kl}^{new}(V, \theta) + \sum_{m \in \Omega} Q_{km}(V, \theta) = 0$$
where:

\( P_{kl}^{\text{new}}, Q_{kl}^{\text{new}} \): active and reactive power flows through the boundary transformer converted to the same base angle of adjacent feeders, respectively;

\( \Omega_{bd}^k \): set of distribution buses adjacent to the boundary bus \( k \).

A zero base angle is the most suitable choice for TS normalization, while \( \phi_{\text{base}} \) is chosen for DFs, as illustrated in Fig. 3. Supposing that the same magnitude of the volt-ampere boundary transformer at the original TS base angle (usually zero) is chosen for the TS and DFs, the active and reactive power flow through the boundary transformer can be converted to the base angle of adjacent feeders by applying Eqs. (8) and (9), that is:

\[
P_{kl}^{\text{conv}} = P_{kl}^{\text{pu}}(V, \theta)\cos \phi_{\text{base}} - Q_{kl}^{\text{pu}}(V, \theta)\sin \phi_{\text{base}}
\]

\[
Q_{kl}^{\text{conv}} = Q_{kl}^{\text{pu}}(V, \theta)\cos \phi_{\text{base}} + P_{kl}^{\text{pu}}(V, \theta)\sin \phi_{\text{base}}
\]

where:

\( P_{kl}^{\text{pu}}, Q_{kl}^{\text{pu}} \): active and reactive power flows through the boundary transformer converted to the base angle of the adjacent DF;

\( P_{kl}^{\text{base}}, Q_{kl}^{\text{base}} \): active and reactive power flows through the boundary transformer to the original TS base angle (usually zero), which are calculated by the conventional power flow equations, i.e.:

\[
P_{kl} = V_k V_l (G_{kl} \cos \theta_{kl} + B_{kl} \sin \theta_{kl}) - G_{kl} V_k^2
\]

\[
Q_{kl} = V_k V_l (G_{kl} \sin \theta_{kl} - B_{kl} \cos \theta_{kl}) + B_{kl} V_k^2
\]

### B. Extended Power Flow Formulation

This section presents the effects of the proposed modifications on the power flow formulation.

Along with the extension of the state vector, shown in Eq. (12), the set of algebraic equations of the power flow, composed by the usual power mismatches, is augmented with the inclusion of the new equations represented by Eqs. (15) and (16), which brings to the problem the representation of the fictitious compensation injections. The resulting augmented set of power flow equations is given by:

\[
f(x) = \begin{bmatrix} \Delta P(V,0,C^p) \\ \Delta Q(V,0,C^q) \\ \Delta PB(V,0) \\ \Delta QB(V,0) \end{bmatrix} = 0 \begin{bmatrix} NPQ \\ NPQ + NPV \\ N_{bd} \\ N_{bd} \end{bmatrix}
\]

where:

NPQ, NPV: numbers of PQ and PV buses, respectively;

\( f(\cdot) \): \( n_x \)-vector function, where \( n_x \) is the number of the extended power flow unknowns, i.e., \( n_x = 2NPQ + NPV + 4N_{bd} \);

\( \Delta P, \Delta Q \): active and reactive power mismatches for each bus calculated according to Eqs. (13-14) and considering the existence of compensation power only at boundary buses;

\( \Delta PB, \Delta QB \): new equations related to transmission and distribution power flows through transmission and distribution lines adjacent to boundary bus, as described in Eqs. (15-16).

The extended power flow problem represented by Eq. (21) can be solved by any traditional power flow method. In this paper, Newton-Raphson’s method was the chosen one. The resulting linear system in this case is given by:

\[
\begin{bmatrix} \Delta P(V,0,C^p) \\ \Delta Q(V,0,C^q) \\ \Delta PB(V,0) \\ \Delta QB(V,0) \end{bmatrix} = \mathbf{J}(x^*) \begin{bmatrix} \Delta \theta \\ \Delta V \\ \Delta C^p \\ \Delta C^q \end{bmatrix}
\]

where \( v \) is the iteration counter and \( \mathbf{J}(x^v) \) is the problem’s Jacobian matrix, given by:

\[
\mathbf{J}(x^v) = \begin{bmatrix} H & N & H_p & 0 \\ M & L & 0 & L_p \\ H_r & N_r & 0 & 0 \\ M_r & L_r & 0 & 0 \end{bmatrix}
\]

The power flow solution can be iteratively obtained by solving the linear system represented in Eq. (21). The state variables are updated at each iteration as:

\[
\begin{bmatrix} \theta \\ V \end{bmatrix}^{v+1} = \begin{bmatrix} \theta^v + \Delta \theta^v \\ V^v + \Delta V^v \end{bmatrix}
\]

\[
\begin{bmatrix} C^p \\ C^q \end{bmatrix}^{v+1} = \begin{bmatrix} C^p + \Delta C^p \\ C^q + \Delta C^q \end{bmatrix}
\]

IV. SIMULATION RESULTS

An extended power flow program was developed based on the formulation described in Section III. In this paper, results obtained for the test-system depicted in Figure 4 are used to illustrate the application of the proposed approach.

The on-line diagram in Figure 4 shows that the network is composed by a transmission network (buses 1 to 5) connected to three distribution feeders (DF1, DF2 and DF3) through three distinct power transformers (2-6, 4-12 and 5-16). Two DG sources are considered at buses 11 (DF1) and 17 (DF3). Buses 6, 12, and 16 are the boundary buses, as illustrated in Figure 4. According to the proposed approach, fictitious active and reactive power compensations are assigned to each boundary bus, as shown in Figure 3. The transmission network and distribution feeder’s data, shown in the appendix, have
been conveniently chosen to ensure convergence even when conventional p.u. normalization is applied.

The simulation results are grouped into three cases, A, B, and C. Case A considers that DFs are radially connected to the TS, that is, an open ring operation among DF1 and DF2 is assumed. Case B, on the other hand, assumes a closed ring operation between DF1 and DF2. Finally, Case C assumes, along with a closed ring operation, that the DG at bus 11 can control its voltage magnitude, that is, bus 11 is treated as a PV type bus.

Furthermore, each case considers a reference case, where null base angle is applied to the whole system, as shown in Table I, which is equivalent to the application of conventional p.u. normalization. A normalized case, which assumes the use of c.p.u. to adequate the X/R ratios of DFs to the same levels of the TS is considered as well. To that end and considering the network line data shown in the appendix, the base angles presented in the third line of Table I are adopted for each part of the network. According to Eq. (6), the application of these base angles in the normalization process allows X/R ratios approximately equal to 4.0 to the entire network.

### TABLE I

<table>
<thead>
<tr>
<th>Base Angles</th>
<th>TS</th>
<th>DF1</th>
<th>DF2</th>
<th>DF3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference Case</td>
<td>0°</td>
<td>0°</td>
<td>0°</td>
<td>0°</td>
</tr>
<tr>
<td>Normalized Case</td>
<td>0°</td>
<td>50°</td>
<td>50°</td>
<td>55°</td>
</tr>
</tbody>
</table>

![Fig. 4: Illustrative T&D test-system](image)

**A. Case A: Open Ring Operation**

This case considers an open ring operation, which implies that all DFs are radially connected to the TS. Following Table I, simulations for Case A are subdivided into cases A0 and A1. Case A0 (the reference case for Case A), assumes null base angles to the TS and DFs. Case A1, in its turn, considers the base angles presented in Table I.

The proposed T&D power flow method is applied to the resulting network. The conventional states (complex voltages at each bus) obtained for Case A1 is exactly the same as the reference case (Case A0), confirming that the proposed approach does not change the power flow solution, as discussed in Section III.

The results regarding power compensation at boundary buses for the reference case are shown in Table II. Since null base angles were adopted for the TS and DFs in this case, the power flows through boundary power transformers (2-6, 4-12 and 5-16) are exactly the same as the power flows injected in their respective DF (represented by power flows through branches 6-7, 12-13 and 16-17). Consistently, active and reactive power compensations are found to be equal to zero for all boundary buses, as shown in the last line of Table II.

Table III presents the power balance at boundary buses for Case A1. It was possible to verify that the power flows through boundary power transformer are exactly the same as in the reference case (see second lines of Tables II and III), since the base angle for the TS is equal to 0° for both cases. As for the values of the power flows through the boundary buses’s adjacent branches (6-7, 12-13 and 16-17) in Case A1 it clearly differs from its values in the reference case. These results are expected, since different base angles had been adopted, as shown in Table I. As one can readily verify the fictitious power injections at boundary buses (C6, C12 and C16), computed as part of the problem solution, are able to compensate the difference between the active and reactive powers from the TS to DFs when no null base angles are employed, as shown in Table III. Finally, active power flows through power transformers indicate that the TS is delivering active power to DF1 and DF2 and receiving active power from DF3, illustrating this new possibility due to the presence of DG sources.

### TABLE II

| Power Balance at Boundary Buses – Case A0 |
|---------------------------------|---|---|---|
| Bus 6  | Bus 12 | Bus 16 |
| 6-2 | -0.1482 | -0.4093 | 0.0976 |
| | -0.1691 | -0.1897 | -0.0508 |
| 6-7 | 0.1482 | 0.4093 | -0.0976 |
| | +0.1691 | +0.1897 | +0.0508 |
| C6 | 0 + j 0 | C12 | 0 + j 0 |
| | | C16 | 0 + j 0 |

### TABLE III

| Power Balance at Boundary Buses – Case A1 |
|---------------------------------|---|---|---|
| Bus 6  | Bus 12 | Bus 16 |
| 6-2 | -0.1482 | -0.4093 | 0.0976 |
| | -0.1691 | -0.1897 | -0.0508 |
| 6-7 | -0.0343 | 0.1178 | -0.0976 |
| | +0.2223 | +0.4335 | +0.0508 |
| C6 | 0.1825 | 0.2915 | 0.0000 |
| | -0.0531 | -0.2458 | +0.1016 |

**B. Case B: Closed Ring Operation**

Case B assumes that the switch connecting DF1 and DF2 is closed. Since DF1 and DF2 are connected to the TS through distinct substations (represented by boundary buses 6 and 12), a meshed topology, as discussed in Section II and illustrated in Fig. 2, is assumed in this case.
As stated in Table I, a reference case considering null base angles for the entire network (Case B0) was simulated, along with Case B1, which considered the adoption of the base angles presented in Table I.

The results regarding the complex voltages at each bus are the same for both cases, as expected. Tables IV and V show the power balance at boundary buses for Cases B0 and B1, respectively. Again, null active and reactive power compensations are found to boundary buses in Case B0, while adequate power compensations values are found in Case B1.

### TABLE IV

<table>
<thead>
<tr>
<th>Bus 6</th>
<th>Bus 12</th>
<th>Bus 16</th>
</tr>
</thead>
<tbody>
<tr>
<td>6-2</td>
<td>-0.3402 -j0.2071</td>
<td>12-4</td>
</tr>
<tr>
<td>6-7</td>
<td>0.3402 +j0.2071</td>
<td>12-13</td>
</tr>
<tr>
<td>C6</td>
<td>0 + j0</td>
<td>C12</td>
</tr>
</tbody>
</table>

### TABLE V

<table>
<thead>
<tr>
<th>Bus 6</th>
<th>Bus 12</th>
<th>Bus 16</th>
</tr>
</thead>
<tbody>
<tr>
<td>6-2</td>
<td>-0.3402 -j0.2071</td>
<td>12-4</td>
</tr>
<tr>
<td>6-7</td>
<td>0.3402 +j0.2071</td>
<td>12-13</td>
</tr>
<tr>
<td>C6</td>
<td>0.2802 -j0.1866</td>
<td>C12</td>
</tr>
</tbody>
</table>

### TABLE VI

<table>
<thead>
<tr>
<th>Bus 6</th>
<th>Bus 12</th>
<th>Bus 16</th>
</tr>
</thead>
<tbody>
<tr>
<td>6-2</td>
<td>-0.3351 -j0.1124</td>
<td>12-4</td>
</tr>
<tr>
<td>6-7</td>
<td>0.1293 +j0.3290</td>
<td>12-13</td>
</tr>
<tr>
<td>C6</td>
<td>0.2059 -j0.2106</td>
<td>C12</td>
</tr>
</tbody>
</table>

C. Case C: GD as a PV bus

As in Case B, this one also considers a closed ring operation among DF1 and DF2. Moreover, it is assumed that the DG source at bus 11 (in DF1) is able to control its voltage magnitude. Therefore, bus 11 is modeled as a PV type bus in the power flow analysis.

Again, the same base angles presented in Table I are adopted. As for cases A and B, the results obtained with the extended power flow for Case C are consistent.

Table VI shows the consistent power balance at boundary buses for Case C1.

### VI. CONCLUSIONS

This paper describes an extended power flow formulation which is able to deal with T&D systems where DFs are explicitly represented in the network model. The proposed methodology is based on the adjustment of X/R ratios of DFs by applying an innovative p.u. normalization, named c.p.u., which considers a complex volt-ampere base. This approach assigns fictitious power injections at T&D boundary buses, which are included into the power flow problem as new state variables. The resulting approach managed to deal with upcoming changes faced by transmission and distributions systems brought by Smart Grid technologies.

Results obtained by applying the method to an illustrative T&D test-system are presented. Different operational conditions, involving radial and meshed topologies for DSs and the presence of DG sources, were selected. The influence of DFs and DG sources onto a TS operation can be accurately determined, since no power injection at T&D boundary buses are previously known. The results validate the proposed methodology and also illustrate the benefits of explicitly representing DFs in the network model.

### VI. APPENDIX

#### TABLE AI

<table>
<thead>
<tr>
<th>Test-system’s Data</th>
<th>Bus Data</th>
<th>P (pu)</th>
<th>Q (pu)</th>
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<tbody>
<tr>
<td>Bus 1</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Bus 2</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Bus 3</td>
<td>0.300</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Bus 4</td>
<td>0.000</td>
<td>0.000</td>
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</tr>
<tr>
<td>Bus 5</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Line Data</th>
<th>R (pu)</th>
<th>X (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>0.040</td>
<td>0.160</td>
</tr>
<tr>
<td>1-5</td>
<td>0.030</td>
<td>0.120</td>
</tr>
<tr>
<td>2-3</td>
<td>0.030</td>
<td>0.120</td>
</tr>
<tr>
<td>3-4</td>
<td>0.080</td>
<td>0.330</td>
</tr>
<tr>
<td>3-5</td>
<td>0.050</td>
<td>0.210</td>
</tr>
<tr>
<td>4-5</td>
<td>0.060</td>
<td>0.250</td>
</tr>
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</table>

#### Distribution Feeder 1

<table>
<thead>
<tr>
<th>Bus Data</th>
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<th>Q (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus 6</td>
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<td>0.000</td>
</tr>
<tr>
<td>Bus 7</td>
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<td>-0.090</td>
</tr>
<tr>
<td>Bus 8</td>
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<td>-0.040</td>
</tr>
<tr>
<td>Bus 9</td>
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<td>-0.015</td>
</tr>
<tr>
<td>Bus 10</td>
<td>-0.050</td>
<td>-0.020</td>
</tr>
<tr>
<td>Bus 11</td>
<td>0.260</td>
<td>0.000</td>
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<table>
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<th>X (pu)</th>
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</tr>
<tr>
<td>6-7</td>
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<td>0.020</td>
</tr>
<tr>
<td>7-8</td>
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<tr>
<td>7-9</td>
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<td>0.020</td>
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<tr>
<td>9-10</td>
<td>0.020</td>
<td>0.020</td>
</tr>
<tr>
<td>10-11</td>
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#### Distribution Feeder 2

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<td>Bus 13</td>
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<td>-0.050</td>
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<tr>
<td>from-to</td>
<td>R (pu)</td>
<td>X (pu)</td>
</tr>
<tr>
<td>---------</td>
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</tr>
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<tr>
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**Distribution Feeder 3**

<table>
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<tr>
<td>19</td>
<td>-0.150</td>
<td>-0.100</td>
</tr>
</tbody>
</table>

**VII. References**


**VIII. Biographies**

Elizete M. Lourenço (M’02) received the Electrical Engineering degree as well as the M.Sc. and Ph.D. degrees in electrical engineering from the Federal University of Santa Catarina, Florianópolis, Brazil, in 1992, 1994, and 2001, respectively. Since 1995, she has been a Faculty member with the Department of Electrical Engineering at the Federal University of Paraná - UFPR, Curitiba, Brazil. Her research interests are in the area of computer methods for power systems operation.

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Odilon L. Tortelli (M’10) received the Electrical Engineering degree as well as the M.Sc. degree in Electrical Engineering from the Federal University of Santa Catarina, Florianópolis, Brazil, in 1992, and 1994, respectively, and the Ph.D. degree in Electrical Engineering from the State University of Campinas, Campinas, Brazil, in 2010. Since 1995, he has been a Faculty member with the Department of Electrical Engineering at the Federal University of Paraná, Curitiba, Brazil. His research interests are in the area of computer methods for power systems operation and control.